CSHPM/SCHPM
Annual Meeting/Colloque Annuel
University of British Columbia
Vancouver, British Columbia

Sunday (2019-06-02):

9:30-9:45 Welcome by Maria Zack (President of CSHPM/SCHPM)(Room: CHEM D213)

Session 1: Early Modern Mathematics
Room: CHEM D213; Presiding: Pat Allaire

9:45-10:15 Eisso Atzema (University of Maine), "Thomas Jefferson and the Shape of the Hyperbolic Paraboloid"

10:15-10:45 Duncan J. Melville (St. Lawrence University), "Commercializing Arithmetic: Educating the Mercantile class in 18th-Century England"

10:45-11:15 Michael Molinsky (University of Maine at Farmington), "The Life (and Library) of Sir Charles Scarburgh"

11:15-11:45 Maria Zack (Point Loma Nazarene University), "Mathematics in Daniele Barbaro's Vitruvius of 1567"

Lunch Break

12:00-14:00 Executive Council Meeting (Room: ANGU 232)

Session 2: History of Mathematics in the Classroom
Room: CHEM D213; Presiding: Amy Ackerberg-Hastings

14:00-14:30 Jonathan Seldin (University of Lethbridge) & Fairouz Kamareddine (Heriot-Watt University), "Using the History of Mathematics to Teach the Foundations of Mathematical Analysis"

14:30-15:00 Dominic Klyve (Central Washington University), "Teaching Statistics from Primary Sources"

15:00-15:30 Po-Hung Liu (National Chin-Yi University of Technology, Taiwan), "History as a Source of Mathematical Narrative"

Monday (2019-06-03):

Session 3a: Proofs & Perception
Room: CHEM D317; Presiding: Dirk Schlimm

9:15-9:45 Zoe Ashton (Ohio State University), "Proof Development, Presentation, and Particular Audiences"

9:45-10:15 Cornelia Knieling (Carnegie Mellon University), "Towards a Feminist Philosophy of Mathematics"

Session 3b: 20th-Century Mathematics
Room: CHEM D200; Presiding: Maria Zack

9:15-9:45 Inna Tokar (City College of New York), "History of Mathematics Education for Gifted Students in the Former Soviet Union. Student Selection Criteria. Diversity of Student Body"

9:45-10:15 Walter Meyer (Adelphi University) **The Silent Critics of Modernism**

Coffee Break

Session 4: Special Session—History of Mathematical Astronomy (Part I)
Room: CHEM D200; Presiding: Craig Fraser

10:30-11:00 Lawrence D'Antonio (Ramapo College of New Jersey), "Clairaut Repeals the Inverse Square Law"

11:00-11:30 Amy Ackerberg-Hastings (Independent Scholar), "Mathematics in Astronomy at Harvard College Before 1839"

11:30-12:00 Craig Fraser (University of Toronto), "Hamilton-Jacobi Theory in Celestial Mechanics 1860-1910"

Lunch Break

12:00-14:00 Annual General Meeting (Room: CHEM D200)

14:00-15:00 Annual Kenneth O. May Lecture: Alexander Jones (New York University), "Sexagesimal Mathematics in Babylonian and Greek Mathematics and Astronomy" (Room: CHEM D200)

Session 4: Special Session—History of Mathematical Astronomy (Part II)
Room: CHEM D200; Presiding: Craig Fraser

15:15-15:45 Glen Van Brummelen (Quest University), "The End of an Error: Ptolemy, Bianchini, and Stellar Coordinates"

15:45-16:15 Kailyn Brooke Pritchard (Quest University), "Determining the Sine Tables Underlying Early European Tangent Tables"

Tuesday (2018-06-04):

Session 5a: Concepts & Abstractions
Room: CHEM D317; Presiding: Greg Lavers

9:15-9:45 Nicolas Fillion (Simon Fraser University), "Concepts of Approximation and the Success of Numerical Methods"

9:45-10:15 Bernd Buldt (Purdue University Fort Wayne), "Abstraction by Parametrization and Embedding: A Contribution to Concept Formation in Modern and Contemporary Mathematics"

Session 5b: New Ideas about Very Old Mathematics
Room: CHEM D213; Presiding: Duncan Melville

9:15-9:45 Daniel Mansfield (University of New South Wales), "Proto-trigonometry and Old Babylonian Land Measurement"

Coffee Break

Session 6a: History of 19th-Century Mathematics
Room: CHEM D213; Presiding: Eisso Atzema

10:30-11:00 Roger Godard (Royal Military College of Canada), "Cauchy, Le Verrier and Jacobi on the Algebraic Eigenvalue Problem and the Secular Variation of Planets"

11:00-11:30 Brenda Davison (Simon Fraser University), "Divergent Series near the Turn of the 20th Century"

11:30-12:00 Fernando Q. Gouvêa (Colby College), "Fruitful Mistakes in Mathematics: The Case of Kurt Hensel"

Session 6b: Realism & Paradoxes
Room: CHEM D317; Presiding: Greg Lavers

10:30-11:00 David Rattray (Simon Fraser University), "The Liar in Context: Revisiting Barwise & Etchemendy's Russellian Solution"

11:00-11:30 Joshua Mozersky (Queen's University), "Human Inquiry, Presupposition, and Natural Structure"

Lunch Break

Session 7: History of Philosophy of Mathematics
Room: CHEM D317; Presiding: Nick Fillion

14:00-14:30 Dirk Schlimm (McGill University), "Historical Views on Good Representations"

14:30-15:00 Christopher Kaumeyer (University of Toronto), "The Three Pillars of Model Theory"

15:00-15:30 Greg Lavers (Concordia University), "Modal Logic and Philosophy of Mathematics in the Mid-Twentieth Century: Carnap, Quine and Barcan Marcus"

15:30-16:00 Molly Kao (Université de Montréal), "Hosiasson-Lindenbaum on Inductive Logic and Analogy"

16:00-16:15 Concluding Remarks (Room: CHEM D317)

End 2019 CSHPM/SCHPM Annual Meeting

ABSTRACTS

Amy Ackerberg-Hastings, Independent Scholar (aackerbe@verizon.net), **Mathematics in Astronomy at Harvard College Before 1839**

While it was by no means a universal phenomenon, mathematics professors in the 18th and 19th centuries in Western Europe and North America often were called upon to teach astronomy. This practice was especially notable and long-lived at Harvard, where John Winthrop (1714–1779) famously took two students to Newfoundland to observe the 1761 transit of Venus and John Farrar (1779–1853) wrote about comets and advocated for the construction of an observatory. The talk will explore the extent to which the astronomy instruction offered by these men and other holders of the Hollis Chair of Mathematics and Natural Philosophy was mathematical, although it will probably focus most on the 1827 translation of Jean-Baptiste Biot's *Traité élémentaire d'astronomie physique* (2nd ed., Paris, 1811), which had Farrar's name on the title page. The establishment of the Harvard College Observatory in 1839 provides a convenient ending point, in part because by then Benjamin Peirce (1809–1880) was also in the process of reshaping the Department of Mathematics.

Zoe Ashton, Ohio State University (ashton.95@osu.edu), **Proof Development, Presentation, and Particular Audiences**

In this paper I will explore the role that proof presentation has in proof development. When the mathematician begins to present his proof as a complete object, it often enters a stage of both evaluation and development. The role that evaluation plays in further development is the core focus of this paper. There is no doubt that the proof itself stands to change based on its evaluation. But in addition to this, I will argue that the mathematician's understanding of acceptable proof is shaped by this process. Objections to steps, definitions, and aspects of her proof can actually inform the way she proves in the future. They stand to inform what she thinks her audience will accept. This gives her a richer imagined audience to work with during proof development. Moreover, I argue that the type of audience which evaluates the proof affects what the mathematician takes from the encounter. Co-authors, mentors, experts, and reviewers all stand to affect the proof and prover in distinct ways. My argument draws on both mathematician's reflections, including Villani's (2015) book, and work in audience-focused argumentation theory.

Eisso Atzema, University of Maine (eisso.atzema@maine.edu), **Thomas Jefferson and the Shape of the Hyperbolic Paraboloid** Although Euler classified the quadric surfaces, there is virtually no trace

of a visual representation of all of the types of quadric surfaces. In particular, as far as I can tell there is no visual representation of the hyperbolic paraboloid in the 18th-century mathematical literature. This is not to say that this type of quadric surface was not studied at all. In fact, the proposed design of the moulding board of a plow by Thomas Jefferson (yes, that one!) involved exactly such a surface. Moreover, it was Jefferson's design that in the early 19th century would inspire Hachette to actually have a model made for this type of quadric surface. In this talk, I will discuss this indirect contribution of Jefferson to the study of quadric surfaces.

Bernd Buldt, Purdue University Fort Wayne (buldtb@pfw.edu), **Abstraction by Parametrization and Embedding: A Contribution to Concept Formation in Modern and Contemporary Mathematics**

The traditional approach to concept formation and definition via abstraction presupposes an Aristotelian ontology and its corresponding hierarchy according to which "definitio fit per genus proximum et differentiam specificam." According to this approach, abstraction is tantamount to removing properties and making the corresponding concept less rich; the more abstract a concept is, the fewer content it has. The traditional approach to abstraction and definition does not, however, provide an adequate model for concept formation and definition in mathematics. What we need instead of the traditional picture is an account of concept formation and definition that is (1) true to mathematical practice; (2) true to the mathematical experience; (3) is compatible with insights from cognitive science. We take this to mean in particular that any such account should be informed by historical case studies (to satisfy (1)) and explain why and how abstract concepts are oftentimes perceived as more powerful and richer, not poorer in content (in order to meet (2)). Requirement (3) needs to be in place for keeping the analysis scientifically sound. Recent accounts of abstraction in mathematics approach the topic by rehashing the development of modern mathematics since the 19th century and, consequently, emphasize aspects such as algebra, set theory, the rise of category theory, or link the development in mathematics to broader cultural shifts. These studies meet to a certain extent (1) and (2). This paper adds to the existing literature by homing in on a topic that lies in the intersection (1) and (2), namely, the question why abstract concepts are perceived as more powerful and richer, not poorer in content. It does so not by tracing any historical developments but by using a number of selected case studies to identify and then discuss various techniques for abstraction that have so far not received proper attention.

Lawrence D'Antonio, Ramapo College of New Jersey (ldant@ramapo.edu), **Clairaut Repeals the Inverse Square Law**

In the 1680s Newton famously tested his inverse square law of gravity on the problem of predicting the orbit of the Moon, but failed. His computation of the precession of the lunar apsides was only half of the observed value. The problem of the Moon's motion lay unsolved but was taken up again by Clairaut, Euler, and d'Alembert in the late 1740s. At a meeting of the Paris Academy of Sciences in 1747, Clairaut announced that both he and Euler had attempted to calculate the rotation of the apsides, but both had found only half of the observed value. At a subsequent meeting of the Paris Academy, Clairaut proposed the replacement of the inverse square law of gravity with a law having both an inverse square and inverse fourth power term. in this talk we will consider Clairaut's failed attempt to predict the motion of the apsides, his abandonment of the inverse square law, the severe reaction of the Newtonians in the Paris Academy (especially that of Count Buffon), and Clairaut's successful computation that correctly predicted the rotation of the apsides and saved the inverse square law.

Brenda Davison, Simon Fraser University (bdavison@sfu.ca), **Divergent Series near the Turn of the 20th Century**

While Euler and others of the mid-18th century had methods for assigning a value to some divergent series, the broad adoption of the Cauchy definition for the sum of a series made such objects problematic. However, by the mid-19th century, renewed interest in these series, as a result of their usefulness in physics, appeared at the hands of Stokes and Poincaré. This talk will examine the reasons for renewed interest in divergent series and the mathematics of summability and asymptotic series that developed in the period from 1880 through 1920, at the hands of Borel and Cesàro, among others, as a result.

Nicolas Fillion, Simon Fraser University (nfillion@sfu.ca), **Concepts of Approximation and the Success of Numerical Methods**

Numerical methods drive contemporary applied mathematics by making it possible to efficiently obtain solutions to problems arising from the study of complex real-world phenomena. However, numerical methods do not generate exact solutions, but inexact ones. Some inexact solutions are deemed approximate, and successful numerical methods are those that generate approximate solutions. There are different ways of precisely characterizing approximate solutions. According to the traditionally predominant practice of applied mathematicians, this is done based on the concepts of perturbation theory and asymptotic analysis. In fact, this is the very concept of approximation that is used, most of the time, to generate the algorithms implemented in numerical methods. Despite this fact, this paper argues that it is not the best way to think about the concept of approximate solution. The argument will be based on a case study, namely, the finite element methods (FEM) used to solve multidimensional systems with irregular boundary conditions. FEMs are among the most dependable computational methods, and they are used with great success. Yet, their more complex discretization scheme fails to be strictly justified in the traditional asymptotic sense, for it commits so-called "variational crime." The paper articulates an alternative, context-dependent concept of approximation that is in line with recently developed methods of a posteriori error analysis and better explains the success of FEMs.

Craig Fraser, University of Toronto (craig.fraser@utoronto.ca), **Hamilton-Jacobi Theory in Celestial Mechanics 1860-1910**

By the second half of the nineteenth century Hamilton-Jacobi theory provided the mathematical tools of choice in celestial mechanics. In this respect celestial mechanics was a very different subject from the one investigated by Lagrange and Laplace a century earlier. We examine the development and consolidation of Hamilton-Jacobi theory in the writings of such figures as Félix Tisserand, Henri Poincaré, E. T. Whittaker and Carl V. L. Charlier. A central development concerned the use of canonical transformations - first invented by Carl Jacobi - in the integration of the canonical equations of motion arising in the three-body problems. The subject as it had coalesced by around 1910 provided mathematical methods that would be taken over and used by German quantum physicists in their investigation of atomic phenomena.

Roger Godard, Royal Military College of Canada (rgodard3@cogeco.ca), **Cauchy, Le Verrier and Jacobi on the Algebraic Eigenvalue Problem and the Secular Variation of Planets**

In this present work, we analyze two numerical approaches on the algebraic eigenvalue problem, the one of the characteristic polynomial by Le Verrier in 1840, and the other one, written by Jacobi in 1846. In 1829, Cauchy introduced the characteristic polynomial of a matrix. The characteristic polynomial will have roots real or complex. Le Verrier's method was designed for the study of the secular variations of planets. It remained for a long time, the method for computing eigenvalues. The computational process comes down to computing successively the powers of matrix then computing their spurs (traces) and solving a recurrent system. Le Verrier's method consisted only in computing the coefficients of the characteristic polynomial. Cauchy and Le Verrier inspired Jacobi, who published in 1846, a powerful but complex method for a real symmetric matrix. In this case, all eigenvalues are real as it was already proved by Cauchy, but Jacobi was able to build an orthogonal system. Here we are interested by his technique of proofs. Therefore, his method was based on a sequence of orthogonal matrices $\{\mathbf{O_k}\}_{\mathbf{k=1}}^{\infty}$ such that $\mathbf{A_{k+1}} = \mathbf{O_k^t A_k O_k} \to \mathbf{D}$ where $\mathbf{D}$ is a diagonal matrix.

Fernando Q. Gouvêa, Colby College (fqgouvea@colby.edu), **Fruitful Mistakes in Mathematics: The Case of Kurt Hensel**

Sometimes a bad mistake can lead to useful insights. When trying to show the mathematical world that p-adic numbers were useful, Kurt Hensel made a spectacular mistake. Fifteen years later, that same mistake led to an important theorem. We will summarize the story and consider both the source of the error and the way it bore fruit.

Alexander Jones, New York University (alexander.jones@nyu.edu), **Sexagesimal Mathematics in**

Babylonian and Greek Mathematics and Astronomy

That sexagesimal (base-60) notation, which survives vestigially in the modern subdivisions of degrees and hours into minutes and seconds, originated in Babylonian mathematics is well known. This lecture will explore the less familiar details of how sexagesimals were differently put to use in Old Babylonian mathematics, Late Babylonian mathematical astronomy, and Greco-Roman astronomy. Two aspects to be highlighted will be the shift from a "floating-point" practice dissociated from units of measure and absolute scale in the Old Babylonian period to the "fixed-point" practice found in the works of the Greek astronomers of Roman times, and the evolution of a consistent notation for zero both as a place-holder and as a null quantity in its own right.

Christopher Kaumeyer, University of Toronto (chris.kaumeyer@mail.utoronto.ca), **The Three Pillars of Model Theory**

The issue of the first result in model theory has led to disagreement in the historical literature, with authors placing its appearance at different points between 1888 and 1915. This lack of consensus suggests that the fundamental principles of model theory and its place in the foundations of mathematics are not entirely understood. Following a description of "the theory of models" given by Tarski in 1954, I isolate what I claim to be the three defining features of model theory, and argue that these aspects came together during a two-fold process involving Emil Post's 1921 completeness theorem and Kurt Gödel's 1930 completeness theorem. I then investigate the views of their historical precedents on the issue of the meaning and use of formal languages, and the way that the above results bring these varied conceptions together. I conclude by noting that the use of model-theoretic techniques requires that philosophers clarify their fundamental principles in a way that captures their development. This issue therefore represents a place where historical analysis can offer insights to philosophers who study the foundations of mathematics and who seek to understand how conceptions of foundations have evolved through time.

Molly Kao, Université de Montréal (molly.kao@umontreal.ca), **Hosiasson-Lindenbaum on Inductive Logic and Analogy**

The foundations of inductive logic are often associated with the name of Rudolf Carnap, among others. More recently, scholars have begun to recognise the important work of Janina Hosiasson-Lindenbaum, a Polish logician who published an earlier account of probabilistic confirmation along with several treatments of paradoxes in the application of this framework to cases of actual reasoning. However, a very infrequently mentioned article is her "Induction et analogie: comparaison de leur fondement" in which she discusses a logical treatment of analogy in addition to the logic of confirmation. I will thus present the comparison of these two systems as they are discussed in this work in order to expand our understanding of Hosiasson-Lindenbaum's system of inductive logic, and to draw some parallels with certain more recent formal treatments of analogy.

Dominic Klyve, Central Washington University (domenic.klyve@cwu.edu),**Teaching Statistics from Primary Sources**

The Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS) project, now in its four year, has worked to develop, disseminate, and study the efficacy of using Primary Source Projects to teach mathematics. Each of these projects uses one or more primary sources to lead students through a standard topic in the undergraduate curriculum. In this talk, we will look at primary sources in statistics the author has written using texts from the 18th, 19th, and 20th century to teach hypothesis testing, p values, regression, and other topics.

Cornelia Knieling, Carnegie Mellon University (cknielin@andrw.cmy.edu), **Towards a Feminist Philosophy of Mathematics**

Science has been criticized on different grounds from feminist perspectives, and there is now an inexhaustible critical literature on feminist philosophy and the nature, purpose, and practice of science.1 While in mathematics feminist critique has long been present, starting from criticizing mathematics as a man's business, its corpus is rather small, mostly concerned with educational studies, and does not reach as far into the mathematical discipline and its methods as its sister does within feminist philosophy of science. Based on the distinction of the context of discovery, the context of justification, and the context of application, I will argue with insights from the philosophy of mathematics and its practice, which show the methodologically diverse, fallible, and socio-cultural character of reasoning in mathematics, that all three contexts can be the focus of a feminist critique. I will then use the rich history of feminist philosophy of science to understand and criticize the interrelation between mathematical research on one side and human life, social relations, and values on the other side to show the effects of social relations and values on mathematical research, and the social aspects of the inquiry itself. The main goal of my talk will be do path a way for a feminist perspective within philosophy of mathematics and to show that a feminist critique of mathematical discovery, problem solving, justification, and application is not only possible but also needed.

Greg Lavers, Concordia University (glavers@gmail.com), **Modal Logic and Philosophy of Mathematics in the Mid-Twentieth Century: Carnap, Quine and Barcan Marcus**

Quine's 'Two Dogmas of of Empiricism' is generally seen a major turning point in the course of analytic philosophy in the twentieth century, one that is particularly concerned with overturning an epistemological picture of mathematics and the sciences. Under the influence of Wittgenstein, Poincaré and others, logical empiricists, including Carnap, had come to see mathematical claims as analytic statements that we could know a priori because they are true no matter how the world happens to be. The majority of the text of 'Two Dogmas...' is devoted to a criticism of the notion of 'analyticity' required by this picture. As a result 'Two Dogmas...' is seen as primarily as response to Carnap (despite Carnap being the target only of two, relatively minor, sub-arguments). What I wish to do in the present paper is stress how these arguments grew out of arguments not having anything to do with large scale epistemological concerns, but as a reaction to Ruth Barcan Marcus's, then recently developed, quantified modal logic. In fact, as I will argue, 'Two Dogmas...' can be seen as growing out of a 1947 paper of Quine's where Barcan Marcus is the explicit target.

Po-Hung Liu, National Chin-Yi University of Technology, Taiwan (liuph@ncut.edu.tw), **History as a Source of Mathematical Narrative**

The stereotypical images for mathematics and narrative are: mathematics is logical, certain, and objective, whereas narrative is often perceived as emotional, indefinite, and subjective. Nonetheless, mathematics and narrative can actually be deeply intertwined. Mathematical narrative is a form of narrative used to communicate or construct mathematical meaning or understanding. To communicate or construct mathematical meaning or understanding, a narrator usually has to introduce some metaphors to induce or promote readers' mathematical understanding. In this talk, I'll address in what way and to what extent history of mathematics can trigger college students' creative mathematical narrative in the courses of "History of Mathematics" and "Mathematics in Ancient Civilizations." Students handed in mathematical fictions created on their own as their final projects. Themes for mathematical fictions are flexible but they were required to use topics which they had learned in the course as the ingredients of their creations. Based on students' mathematical fictions, this talk will discuss (1) the relationship between literary narrative and mathematical narrative, and (2) the role of history in developing students' mathematical narrative.

Daniel Mansfield, University of New South Wales (daniel.mansfield@unsw.edu.au), **Proto-trigonometry and Old Babylonian Land Measurement**

Several school texts from the Old Babylonian period (1900-1600 BCE) contain evidence of an early numerical understanding of the fundamental relationship between the sides of a right triangle - known today as Pythagoras' theorem. In 2009 Robert Adams suggested that this interest "had something to do with the increasing need for cadastral accuracy". In 2017 Mansfield and Wildberger went further and claimed that Plimpton 322, the most famous list of Old Babylonian Pythagorean triples, was a proto-trigonometric table that could have been used for land measurement. This talk examines the social and mathematical evolution the land measurement from the Ur III period (2100-2000 BC) to the Old Babylonian period. New evidence will be examined, along with a revised interpretation as to the possible connection between Plimpton 322 and land measurement.

Duncan J. Melville, St. Lawrence University (dmelville@stlawu.edu), **Commercializing Arithmetic: Educating the Mercantile Class in 18th-Century England**

The expansion of trade in England in the late 17th and early 18th centuries generated a demand for a class of numerate clerks, merchants and accountants. Publishers and authors responded with new products for educating young merchants and aiding in their daily computations. Among the most successful of these authors was Edward Hatton (1664—1737). We shall survey his diverse output and explain the strategies he pursued in support of the arithmetic needs of the emerging mercantile class.

Walter Meyer, Adelphi University (meyer1@adelphi.edu) **The Silent Critics of Modernism** In the

late 19th century, mathematical research started to become heavily influenced by a philosophy that has been called modernism. Modernism is characterized by an emphasis on abstraction, rigor, foundations of mathematics and a disinterest in applications (see Gray's *Plato's Ghost*). This was a compelling approach to mathematics in the United States at the same time, propelled by E. H. Moore, O. Veblen, E. V. Huntington, R. L. Moore and G. A Miller, etc.

Judging by the slow pace with which undergraduate courses in Abstract Algebra were adopted in the United States, it is clear that there was resistance among undergraduate instructors. Not until midcentury did these courses become omnipresent. We shall demonstrate this through a sample of 40 different undergraduate institutions.

Skeptics left little in the written record to explain their stances. Nonetheless, we shall consider what factors might have supported their resistance.

During the era we discuss, strains became apparent between research and teaching in the United States. A well-known example of this is the formation of the breakaway organization, MAA, oriented firmly to undergraduate matters. Our story of the slow adoption of Abstract Algebra is another example.

Michael Molinsky, University of Maine at Farmington (michael.molinsky@maine.edu) **The Life (and Library) of Sir Charles Scarburgh**

Although perhaps better known for his scholarly work in medicine and his service as physician to royalty, Sir Charles Scarburgh (or Scarborough) had a lifelong interest in mathematics, culminating in a new English translation of the first six books of Euclid's Elements that was published posthumously by his son, Edmund. This talk will briefly examine the life of Sir Charles Scarburgh, his relationships with other contemporary mathematicians, and some of the contents of his vast mathematical library.

Joshua Mozersky, Queen's University (mozersky@queensu.ca), **Human Inquiry, Presupposition, and Natural Structure**

An essential component of external, or metaphysical, realism is the thesis that the world is, largely, independent of human thought and inquiry. A certain sort of antirealism argues that, on the contrary, what there is depends on, or is relative to, human investigation: there is no way the world is independent of our ways of describing it. In this presentation I argue that the very fact that we engage in some kinds of inquiry entails that at least some structure exists independently of all inquiry. In short, the existence of inquiry supports the independence portion of realism; the assumption that what there is depends upon human inquiry is, in the end, self-undermining. I end with an examination of the significance of this conclusion by comparing it to the views of some of the most prominent philosophers of science of the past half century.

Kailyn Brooke Pritchard, Quest University (kailyn.pritchard@questu.ca), **Determining the Sine Tables Underlying Early European Tangent Tables**

Since its initial appearance in Europe in the 1400's, the tangent function has evolved from a useful auxiliary function into one of the central functions in trigonometry. Tangent tables were used exclusively in the fields of astronomy and spherical geometry until late in the 16th century, when their functionality was extended to earthly pursuits such as surveying and architecture. This presentation will discuss my analysis of some of the earliest tangent tables known to have been produced in Europe, including works by historical actors such as Giovanni Bianchini(1410-1469), Regiomontanus (1436–1476), and Georg Rheticus (1514–1574). By working from the tangent tables, I have been able to determine the methods by which the tables were computed, identify the radii of the underlying sine and cosine tables (as the unit circle was not yet conceived of), and reconstruct the underlying sine and cosine tables themselves. The results are surprising; the underlying tables aren't tables we have seen before. The discrepancies between the published tables and those they used to complete their computations demonstrate that the mathematicians and astronomers creating these tables continually sought to improve their work.

David Rattray, Simon Fraser University (drattray@sfu.ca), **The Liar in Context: Revisiting Barwise & Etchemendy's Russellian Solution**

John Barwise and John Etchemendy (1987) propose a contextualist solution to the Liar paradox based on situation semantics and non-wellfounded set theory. They maintain that their "Austinian" approach is better than a traditional "Russellian" approach to the paradox. In this paper, I argue that their account is implicitly question-begging, which motivates revisiting the Russellian account. I then improve on the Russellian account by using a model-theoretic technique developed by Feferman and Aczel (1980). Specifically, I construct a "closed-off" Russellian model based on Feferman's Fixed Point Theorem (1984). I hope to show that this modified Russellian solution upholds Barwise and Etchemendy's intuitions about truth, while avoiding the challenge facing their favoured "Austinian" account.

Dirk Schlimm, McGill University (dirk.schlimm@mcgill.ca), **Historical Views on Good Representations**

Notations are pervasive in mathematics, but what makes a notation "good"? I will present some criteria for good mathematical and logical notations that have been put forward by leading 19th century logicians, such as Babbage, (1830), Boole (1854), Frege (1879), and Peirce (1885). From this discussion it will emerge that some of the criteria are incompatible with each other and that what counts as "good" can only be evaluated relative to a particular purpose or goal one wants to achieve with the representation. These goals can be theoretical (e.g., minimizing the number of symbols), practical (e.g., allowing for easy inferences), pedagogical and cognitive (e.g., easy to learn and memorize).

Jonathan Seldin, University of Lethbridge (jonathan.seldin@uleth.ca) (with Fairouz Kamareddine, Heriot-

Watt University (f.d.kamareddine@hw.ac.uk)), **Using the History of Mathematics to Teach the Foundations of Mathematical Analysis**

At last year's meeting of the CSHPM, we presented the first paper of this title. We have been writing a book on this, the main idea of which is to avoid the use of what Keith Devlin in 2005 called "formal definitions," which are definitions that nobody can understand without working with them. For students without mathematical maturity, these definitions can be difficult to learn from. The tentative title of our book is *A Non-Formal Introduction to Mathematical Analysis*.
The main idea we took from history was to use the ancient Greek method of exhaustion to introduce the $\epsilon - \delta$ and $\epsilon - N$ definitions of limits, along the lines of Seldin's paper "From exhaustion to modern limit theory," presented to the Society in 1990.
Some feedback we have received from our initial draft of the book suggested that we might combine this book with a book to precede this one on the bridge from computational courses (such as calculus) to advanced mathematics, and make the entire thing a book that could be used for a two-semester course for such a bridge.

Inna Tokar, City College of New York (innatokar@gmail.com), **History of Mathematics Education for Gifted Students in the Former Soviet Union. Student Selection Criteria. Diversity of Student Body**

This presentation will continue to examine programs for mathematically talented students in the former Soviet Union. Special emphases will be given to the boarding schools for gifted students at Moscow, Novosibirsk, St. Petersburg and Kiev Universities. The origins and history of education for gifted students in the former Soviet Union will be discussed. Specifically, the following questions will be considered:

1. What is the nature of and variations among special school curricula and student, faculty, and alumni bodies?

2. Student selection criteria and admissions policies. Diversity of student body.

To answer these questions, original literature from Russia and Ukraine was reviewed, including scientific publications, educational journals, government and university documents. Interviews were conducted with Soviet-born mathematicians and educators who created and taught at these schools.

Glen Van Brummelen, Quest University (glen.vanbrummelen@questu.ca), **The End of an Error: Ptolemy, Bianchini, and Stellar Coordinates**

In the Almagest, Ptolemy's treatment of the problem of the conversion of stellar coordinates contained a mathematical lacuna. This gap, transmitted via al-Battānī to medieval Europe, led to a series of errors by a number of astronomers through the 14th century. The mistake was finally remedied in the Tabulae primi mobilis by 15th-century Italian astronomer Giovanni Bianchini; one of the tables he composed to solve the problem turned out to be the birth of the tangent function in Europe. We explore the history of the stellar coordinates problem, and conclude by portraying Bianchini's work on this topic as the source of Regiomontanus's renowned Tabulae directionum.

Maria Zack, Point Loma Nazarene University (mzack@pointloma.edu), **Mathematics in Daniele Barbaro's Vitruvius of 1567**

Marcus Vitruvius Pollio, known as Vitruvius, was a first-century BC Roman architect and engineer best known for his work *De architectura*. This multi-volume text discusses standard architectural forms in the ancient world. Vitruvius' work was "rediscovered" in the sixteenth century and was one of the books that fed the interest of symmetry and classical forms in European architecture. One interesting translation of Vitruvius was done by Daniel Barbaro who translated *De architectura* into Italian in 1567. Vitruvius' original text contained almost no calculations, however Barbaro introduced mathematics into his text. This talk will look at some of the interesting mathematics in Barbaro's version of Vitruvius.