CSHPM/SCHPM
Annual Meeting/Colloque Annuel
University of Calgary
Calgary, Alberta
Final Program

Sunday (2016-05-29):

9:30-9:45 Welcome by Dirk Schlimm (Vice-President of CSHPM/SCHPM)

Session 1: Philosophy of Mathematicians
Room: Science Corridor B-142; Presiding: Richard Zach

9:45-10:15 Jeremy Shipley (Harper College), "Poincaré on the Foundations of Geometry in Understanding"

Coffee Break

Session 2: Basic Notions in Geometry
Room: Science Corridor B-142; Presiding: Lawrence D'Antonio

10:30-11:00 Amy Ackerberg-Hastings (University of Maryland University College), "John Playfair and his Misnamed Axiom"

11:00-11:30 Maritza Branker (Niagara University), "A comparison of Cauchy and Hamilton's treatment of complex numbers"

11:30-12:00 Sandra Visokolskis (National University of Cordoba), "Greek Geometrical Analysis and a Plausible Oriental Source in the Method of Single False Position: A Discussion"

Lunch Break

12:00-14:00 Executive Council Meeting (Room: Social Sciences 1253)

Session 3: Special Session on Logic and Mathematics in the 19th and 20th Centuries in Honour of Aldo Antonelli
Room: Science Corridor B-142; Presiding: Dirk Schlimm/Richard Zach

14:00-14:10 Welcome

14:10-14:35 Dirk Schlimm (McGill University), "Frege's Begriffsschrift Notation: Design Principles and Trade-offs"

14:35-15:00 Rachel Boddy (University of California Davis), "Reconsidering Frege's View of the Foundation of Arithmetic"

15:00-15:25 Aaron Thomas-Bolduc (University of Calgary), "Between Logicism and Neo-Logicism"

15:25-15:50 Richard Zach (University of Calgary), "The Decision Problem and the Model Theory of First-order Logic"

Coffee Break

16:00-16:25 Teppei Hayashi (University of Calgary), "Categorical Interpretation of Peirce's Continuum"

16:25-16:50 Edward Shear, Jonathan Weisberg, Branden Fitelson (University of California Davis), "Two Approaches to Belief Revision"

16:50-17:15 Jonathan P. Seldin (Lethbridge University), "Some Philosophical Results on Incompleteness"

Monday (2016-05-30):

Session 4a: Biography (Parallel Session)
Room: Kinesiology B-131; Presiding: Amy Ackerberg-Hastings

9:15-9:45 Henryk Fukś (Brock University), "Open problems from the 17th century: Adam Adamandy Kochański and his mathematical works"

9:45-10:15 George Heine (Independent Scholar), "Mathématiques: Une Promenade Parisienne"

Session 4b: Philosophy of Mathematicians (Parallel Session)
Room: Kinesiology B-133; Presiding: Dirk Schlimm

9:15-9:45 André Curtis-Trudel (University of Calgary), "Is Church' Thesis an Explication?"

9:45-10:15 Paul McEldowney (University of Notre Dame), "Bolzano against Kant's Pure Intuition"

Coffee Break

Session 5a: History of Mathematics in a New Light (Parallel Session)
Room: Kinesiology B-131; Presiding: Patricia Allaire

10:30-11:00 Roger Godard (Department of National Defense), "A Convolution on the Convolution as a Mathematical Tool"

11:00-11:30 Rob Bradley (Adelphi University), "Polar Ordinates in Bernoulli and L'Hôpital"

11:30-12:00 Joel Silverberg (Roger Williams University), "Napier, Torporley, & Menelaus — A closer look at Augustus De Morgan's observations on early Seventeenth-century restructuring of planar and spherical trigonometry"

Session 5b: Mathematical Logic (Parallel Session)
Room: Kinesiology B-133; Presiding: Jonathan Seldin

10:30-11:00 ~~William D'Alessandro (University of Illinois-Chicago), "Intertheoretic Reduction and Explanation in Mathematics"~~ (Cancelled)

11:00-11:30 Matthias Jenny (MIT), “The ‘If’ of Relative Computability”

11:30-12:00 Michael Cuffaro (University of Western Ontario), “Quantum Reflections on Computational Complexity”

Lunch Break

12:00-14:00 Annual General Meeting (Room: Kinesiology B-133)

14:00-15:00 Annual CSHPM Kenneth O. May Lecture: James Tappenden (University of Michigan), “Frege, Carl Snell and Romanticism; Fruitful Concepts and the ‘Organic/Mechanical’ Distinction” (Room: Kinesiology B-133)

Session 6: 18th-Century Mathematics
Room: Kinesiology B-133; Presiding: Robert Bradley

15:15-15:45 Eisso Atzema (University of Maine), “Lexell on Spherical Geometry”

15:45-16:15 David Bellhouse (University of Western Ontario), “The Case of the Laudable Society for the Benefit of Widows”

16:15-16:45 Lawrence D’Antonio (Ramapo College of New Jersey), “‘A Debate over Words’: d’Alembert and the Vis Viva Controversy”

17:00-19:00 President’s Reception

Tuesday (2016-05-31):

Session 7: Objects Mathematical and Otherwise
Room: Kinesiology B-133; Presiding: V. Frederick Rickey

9:45-10:15 Valerie Allen (John Jay College of Criminal Justice, CUNY), “What is a Symbol?”

Coffee Break

Session 8a: Mathematical Logic (Parallel Session)
Room: Kinesiology B-133; Presiding: Sylvia Svitak

10:30-11:00 Fabio Lampert (University of California, Davis), “Actually, Tableaux, and Two-Dimensional Logic”

11:00-11:30 Corey Mulvihill (University of Ottawa), “Proofs of intermediate logics from Intuitionistic Logic plus Epsilon and the Ontological Status of Multivalent Concepts”

Session 8b: History of Modern Algebra (Parallel Session)
Room: Kinesiology B-131; Presiding: Craig Fraser

10:30-11:00 Janet Barnett (Colorado State University at Pueblo), “Is the Disjunctive Form Really Normal? Teaching Boolean Algebra via Original Sources”

11:00-11:30 Fernando Gouvêa (Colby College), “The Mystery of the Extra Divisors”

Lunch Break

Session 9: Presenting Mathematics in the 20th Century
Room: Kinesiology B-133; Presiding: Eisso Atzema

14:00-14:30 Craig Fraser (University of Toronto), "Mathematics Subject Classification — 1880 to Present"

14:30-15:00 Mariya Boyko (University of Toronto), "Mathematical School Reforms in Post-War America and the Soviet-Union: A Comparative Study"

Coffee Break

Session 10: Euler
Room: Kinesiology B-133; Presiding: Eisso Atzema

15:15-15:45 William Hackborn (University of Alberta, Augustana Campus), "Euler's Method for Computing the Movement of a Mortar Bomb"

15:45-16:15 V. Frederick Rickey (West Point), "E228"

16:15-16:30 Concluding Remarks (Room: Kinesiology B-133)

End 2016 CSHPM/SCHPM Annual Meeting

We gratefully acknowledge the generous support of the Calgary site office of the Pacific Institute for the Mathematical Sciences and of the Department of Philosophy at the University of Calgary.

Pacific Institute for the Mathematical Sciences

UNIVERSITY OF CALGARY
FACULTY OF ARTS
Department of Philosophy

ABSTRACTS

Amy Ackerberg-Hastings (University of Maryland University College), **John Playfair and His Misnamed Axiom**

The term "Playfair's Axiom" is a mainstay of school geometry textbooks as well as one of the few things many mathematicians know about John Playfair (1748-1819), Professor of Mathematics and then Natural Philosophy at the University of Edinburgh. However, the ubiquity of the phrase masks considerable historical complexity. At least three different versions of the statement circulate—only two of which appeared in the editions of Playfair's Elements of Geometry—while the underlying concept dates back to Proclus. Recent research has revealed new information about when and why "Playfair's Axiom" became commonplace as a label. I will also update work that appeared in the 2007 Proceedings about logical differences between the versions of the statement and about the history of Playfair's interest in parallels.

Valerie Allen (John Jay College of Criminal Justice, CUNY), **What is a Symbol?**

Algebraic symbols exceed mere notation and abbreviation to allow abstraction and calculations that at the levels of common sense and counting numbers seem counterintuitive. Basic examples include negative numbers and their operations—e.g. (-3).(-7)=21. As Marin Mersenne observed in 1625, such a product appears to be contrary to all reason ("semble estre contre toute sorte de raison"). By this definition of symbolism, the choice of signifiers used (whether words, ligatures, special notation, or abbreviations) seems of secondary importance. In fact, one may perform a symbolic operation with words or a non-symbolic one with notation. Yet the very period in which algebra emerged as a fully symbolic language—C16th to C18th—is one marked by the instability of its notation, the need for and resistance to it, the impossibility of consensus, and conflicted opinions of its elegance or ugliness. Of the notation of mathematician John Wallis, philosopher Thomas Hobbes remarked that the characters on the page looked "as if a hen had been scraping there." The devil lies in those details of a mathematical idea articulating itself in a language that is beyond all vernaculars yet immanent in each. The contingent conditions in which notation emerged as an agreed script, being particular yet universal, are little catalogued, yet in their exposure lies a rich story of the struggle with form. The interest in algebra's textual practice during this period bears comparison with the discussions of symbol in poetic theory of the Romantic era—especially Goethe and S.T. Coleridge. For Coleridge, symbolic expression attains something like the synthetic power of reason in Hegelian dialectic, which unifies apparent contradictions into a higher totality. Although poetic and mathematical symbols do very different kinds of work, both are at odds with the phenomenal world yet at the same time fundamentally redescribe it.

Eisso J. Atzema (University of Maine), **Lexell on Spherical Geometry**

In this talk I will discuss some of Lexell's work in spherical geometry and its connection to the work of Euler and others of around the same time. Particularly, I will focus on what is sometimes called Lexell's Theorem, stating that the locus of the vertices of all spherical triangles with a common base and equal area is a small circle. My main point of reference will be the recent Lexell biography by Johan Sten and Athanase Papadopoulos' long article on Euler and spherical geometry that appeared last year. The main question that I will seek to answer is whether the work of Lexell and others on spherical geometry should be viewed in the context of traditional spherical trigonometry (and by extension as a chapter in the history of non-Euclidean geometry) or whether it perhaps is best viewed as an expression of a first serious interest in spatial geometry and the geometry of surfaces.

Janet Barnett (University of Colorado at Pueblo), **Is the disjunctive form really normal? Teaching boolean algebra via original sources**

In 1847, George Boole launched the study of boolean algebra with a bold new approach to logic. He further developed this approach in his 1854 *An Investigation of the Laws of Thought.* Although few copies sold when it first appeared (Boole and a friend who bore the expense of its initial printing probably did not recover their costs), the algebraic methods of Laws of Thought attracted considerable attention in ensuing years. Axiomatized by Edward V. Huntington in 1904, the abstract structure known today as a boolean algebra was eventually recognized as an important tool in applications outside of mathematics, most notably in Claude Shannon's groundbreaking work on circuit design of 1938. Boolean algebra remains important today, as both an interesting mathematical object in its own right and a powerful tool for applied practitioners.
This talk surveys the (hi)story of Boole's 'Algebra of Logic' through a sequence of three student projects based on original source readings from Boole, Huntington, Shannon and others. We then focus on extracts from the third project that introduce the concept of disjunctive normal form through excerpts from Boole and Shannon. Experiences using these projects with both undergraduates and advanced middle school students will be shared.

David Bellhouse (University of Western Ontario), **The Case of the Laudable Society for the Benefit of Widows**

A turning point in the history of actuarial science is the bursting of the Annuity Bubble in the 1770s. In the previous decade more than a dozen societies had formed in London, England to make life annuity payments to a beneficiary. Without a proper actuarial foundation, none of these societies was properly funded. The situation was widely exposed in 1771 with Richard Price's publication of Observations on Reversionary Payments. Price's influence was such that most of these annuity societies ceased operation and the remaining ones tried to reform their premium and benefit structures. I will focus on the oldest of these societies, the Laudable Society for the Benefit of Widows. The reform of this society resulted in debates and decisions in the British House of Commons and involved four mathematicians from various walks of life: James Horsfall (FRS, barrister at the Middle Temple), Richard Price (FRS, political philosopher and Nonconformist minister), Edward Waring (FRS, Lucasian Professor of Mathematics), and Benjamin Webb (accountant and master of a grammar school). This is but one episode in the book I am writing, The Emergence of Actuarial Science in Eighteenth Century England: Mathematicians and Life Contingent Contracts.

Rachel Boddy (University of California Davis), **Reconsidering Frege's view of the foundation of arithmetic**

To date, there continues to be disagreement over Frege's motivation for proving the basic propositions of arithmetic. Traditionally, Frege's logicist project is seen as a search for epistemic foundations—viz. to identify the primitive truths that ground the justification for our knowledge of arithmetic. The problem through which Frege introduces his project in Foundations, however, is that arithmetic lacked a well-understood subject matter and hence also a proper scientific foundation and in Basic Laws he argues that numbers are logical objects. Both have been neglected by the traditional interpretations of Frege's motivations. In this paper, I discuss an overlooked and more consistent motivation for Frege's logicist project, which appears to be preempted by the common assumption that Frege thought of the foundations of arithmetic as a set of primitive truths. The thesis I develop is that Frege thought arithmetic lacked a scientific foundation as long as it lacked a definition of number, and that logicism was part of Frege's defense of his definition. In particular, I show that Frege thought that he needed to prove the basic propositions of arithmetic in order to confirm his definition of number.

Mariya Boyko (University of Toronto), **Mathematical School Reforms in Post-War America and the Soviet Union: A Comparative Study**

North American historians of mathematics education have provided detailed accounts of the 1960s "new mathematics" movement, its goals, features and aftermath. Parallel to the reforms in the West, but somewhat later, innovative and fundamental changes to mathematics education were also being carried out in the Soviet Union. Soviet educational theorists were aware of the Western developments and discussed them in periodicals devoted to mathematics education. The Soviet reforms and their lasting legacy have not been covered adequately in the literature thus far. The paper will examine aspects of these reforms and provide a comparison of the Russian experience with what took place in the West.

Robert Bradley (Adelphi University), **Polar Ordinates in Bernoulli and L'Hôpital**

According to the usual narrative, priority for the invention of polar coordinates belongs to Newton, although Jakob Bernoulli has priority of publication in 1691, because Newton's results were only published posthumously, almost forty years later. However, it was not until the middle of the 18th century that polar coordinates took on a form that would be recognized by today's readers. Earlier versions all featured ordinates emanating from a single point or pole, with some geometric construction playing the role that now belongs to an angular coordinate. The largest and most accessible collection of these early schemes of polar ordinates is probably to be found in l'Hôpital's *Analyse des infiniment petits* (1696), based on the lessons given to the Marquis by Johann Bernoulli. In this talk, I will describe Bernoulli's approaches to polar ordinates, as presented in l'Hôpital's textbook.

Maritza Branker (Niagara University), **A comparison of Cauchy and Hamilton's treatment of complex numbers**

Cauchy's *Cours d'analyse* was published in 1821 and consisted of his lecture notes on the burgeoning field of complex analysis. William Rowan Hamilton published his formal treatment of complex numbers in the extensive treatise, *Theory of Conjugate Functions, or Algebraic Couples; with a Preliminary and Elementary Essay on Algebra as the Science of Pure Time.* We will discuss the difference in the use of analogy to convince their audience of the validity of complex numbers as mathematical objects. The use of analogy in Hamilton's work was discussed in the article, "Analogy in William Rowan Hamilton's New Algebra" by Maritza M. Branker and J. Little, (*Technical Communication Quarterly* Vol 21, Issue 4 (2012), pp. 277-289). This talk represents a continuation of this line of research, focusing on the elegant approach taken by Cauchy.

Michael Cuffaro (Ludwig Maximilian University, Munich), **Quantum Reflections on Computational Complexity**

Computational complexity theory is a branch of computer science that is dedicated to classifying computational problems in terms of their intrinsic difficulty. While traditional computability theory tells us what we can compute in principle, complexity theory informs us with regard to our practical limits. Computational complexity thus provides a bridge between the philosophy of mathematics and other areas such as the philosophy of technology and the philosophy physics. Within the latter, the science of quantum computation invites us to consider quantum physical systems as computational resources. It turns out that the essential difference between classical and quantum systems, from this point of view, manifests itself in how difficult it is (complexity-theoretically) to accomplish certain tasks. In this talk, I argue that considering quantum computation also illuminates our understanding of complexity-theoretic concepts themselves, in that it emphasises that complexity theory cannot aspire to be a model-independent science such as computability theory. At the same time, I will argue that this does not, as some have suggested, force us to revise our complexity-theoretic concepts. For although this aspect of complexity theory is obscured by scientific practice, it is nevertheless implicit in the framework provided by the theory.

André Curtis-Trudel (University of Calgary), **Is Church's Thesis an Explication?**

Church's thesis claims that the precise notion of recursivity captures the imprecise concept of effective calculability. Many regard the thesis as true, but unprovable. On this view there is an open question about the nature of the thesis: if it is unprovable, and hence not a theorem, what kind of claim is it? A recent answer - which I call 'explicationism' - suggests that the thesis is an explication, in the sense introduced by Carnap. But despite its attractiveness, explicationism has yet to receive a satisfactory treatment. Thus my first aim is to give explicationism a much needed precise characterization. Once characterized, I assess explicationism and find it wanting. Correspondingly, my second aim is to argue that explicationism, while enticing, should be rejected. In particular, I argue (a) that one can adopt explicationism only by abandoning important and widely accepted uncomputability results, and (b) that it is a general constraint that any account of the nature of Church's thesis should preserve such results. Explicationism fails to satisfy the general constraint and so is unsatisfactory as a theory of the nature of Church's thesis.

William D'Alessandro (University of Illinois-Chicago), **Intertheoretic Reduction and Explanation in Mathematics**

Intertheoretic reduction has been a central theme in the philosophy of science since Ernest Nagel drew attention to the notion some fifty years ago. And rightfully so. Reduction is a widespread and important phenomenon, and by better understanding it we stand to gain valuable insights about the metaphysics, epistemology, psychology and practice of the empirical sciences.

Reduction is widely acknowledged to occur in pure mathematics also. (Everyone is familiar with the idea that we can "reduce mathematics to set theory," for instance.) Nevertheless, relatively little attention has been paid to the above sorts of questions as they arise in the mathematical context. This is unfortunate and rather unaccountable, since the answers to such questions might well prove as illuminating for the philosophy of mathematics as their counterparts have for the philosophy of science.

I try to address one aspect of this imbalance by investigating the relationship between reduction and explanation in the mathematical setting. I argue for three claims: (1) Intertheoretic reduction in mathematics should be understood in broadly Nagelian terms. That is, it should be understood as an essentially linguistic phenomenon that need not involve identity, composition or other metaphysical relations. (2) Intertheoretic reductions are relatively common and natural in mathematics, just as they are in empirical sciences. (3) Unlike what appears to be the case in the empirical context, where a successful reduction is virtually always an explanatory achievement, only some mathematical reductions are explanatory.

Lawrence D'Antonio (Ramapo College of New Jersey), **Title: "A Debate over Words": D'Alembert and the Vis Viva Controversy**

The vis viva controversy concerned the nature of force and its proper measure. Starting with Descartes' definition of force as the quantity of motion, through Leibniz' counterproposal of vis viva, the controversy became a focal point of discussion in the Berlin Academy of Science, the Paris Academy of Science, and the Royal Society. This talk will focus on how the debate was structured around the metaphysical inclinations of the disputants. D'Alembert attempted to end this controversy by labeling it "une dispute des mots." We will consider whether d'Alembert was successful in ending the debate.

Craig Fraser (University of Toronto), **Mathematics Subject Classification 1880 to Present**

Looking at library and journal classifications of mathematical subjects, we examine how different parts of mathematics were understood to be related and how this understanding changed over time, as old classifications were modified and new classifications were created. Library classification systems of interest include the Dewey Decimal System and the Library of Congress System, as well as the classifications used by the British Library. Also of note are the classifications that were adopted by such projects as the *Encyklopädie der mathematischen Wissenschaften*, and by the abstracting journals *Jahrbuch über die Fortschritte der Mathematik*, *Zentralblatt für Mathematik und ihre Grenzgebiete*, and *Mathematical Reviews*, among others. The talk is a preliminary report.

Henryk Fukś (Brock University), **Open problems from the 17th century: Adam Adamandy Kochański and his mathematical works**

Adam Adamandy Kochański SJ (1631–1700) was a Polish Jesuit mathematician, inventor, and polymath. His interest were very diverse, including problems of geometry, mechanics and astronomy, design and construction of mechanical clocks and computing machines, as well as many other topics. He published relatively little, and most of his mathematical works appeared in the *Acta Eruditorum* between 1682 and 1696. His most interesting two papers deal with the problem of rectification of a circle and construction of a novel type of magic squares. On both of these subjects, Kochański presented some intriguing results without proving them, and also posed several open problems. The talk will describe author's efforts to reconstruct Kochański's reasoning and to solve some open problems he posed. Colorful historical background of Kochański mathematical endeavors will be discussed as well.

Roger Godard (Department of National Defense), **A convolution on the convolution as a mathematical tool**

For many scientists, the convolution is a fundamental tool of applied mathematics from the XXth century. However, the discrete convolution goes back to the Chinese Middle-Ages mathematics. The convolution integrals appeared during the XVIIIth century with Euler for ODE, and the beginning of the XIXth century in the potential theory, the heat conduction equation, and the wave equation with Cauchy, Fourier, Laplace, and Poisson. They also appeared from trigonometric operations as a convenient way to represent analytical results with Poisson and Dirichlet. From Christaan Huygens, Clairault, Lagrange, Laplace, Poisson, Liouville, Dumanel, Boltzmann, Volterra, we see the emergence of a mathematical tool for physical processes about memory, time-delay and superposition of events. We shall present some applications of the convolution in the theory of probability and data processing. Finally, we consider the modern mathematical properties of the convolution with Volterra, Lebesgue, Doestz, Weyl, Weil and L. Schwartz.

Fernando Q. Gouvêa (Colby College), **The Mystery of the Extra Divisors**

In the 1870s, Dedekind proved a fundamental theorem describing how certain prime numbers factored when one extended the integers to more general rings of algebraic integers. The theorem applied to primes that satisfied a precise condition. Having raised the question whether it was possible that every prime number satisfied that condition, Dedekind immediately realized that there were examples where this was false. Determining when and why this happened became known as the problem of the "common inessential discriminant divisors." We will explain the problem, discuss Hensel's early work and eventual solution, and explore the implications of that solution. This talk is a preliminary report on joint work with Jonathan Webster.

William W. Hackborn (University of Calgary), **Euler's method for computing the movement of a mortar bomb**

This paper addresses Euler's efforts to "determine the movement of a bomb, or of a cannonball," published in Memoirs of the Berlin Academy of Sciences, 1755 (E217). Euler begins E217 by deriving a solution—first found and expressed using quadratures more than three decades earlier by Johann Bernoulli—for the trajectory of a projectile subject to uniform gravity and a drag force proportional to an arbitrary power of its speed, and he then uses this solution to construct a numerical method with which the significant attributes (e.g. range, time of flight) of a mortar shot can be calculated by artillery officers on the battlefield. Typically a mortar bomb, intended to fly over enemy fortifications and explode in the air a short distance above its target, is shot at a relatively steep angle and low (subsonic) speed; in this case, the air resistance on the bomb is roughly proportional to the square of its speed, and Euler's method incorporates this fact.

Teppei Hayashi (University of Calgary), **Categorical Interpretation of Peirce's Continuum**

Peirce had a rather peculiar conception of a continuum, especially later in his career. He claimed that the real number system R is not enough to represent or capture the real continuum; in order to make $\mathbb{R}$ continuous, according to Peirce, we need to put some (actually, infinitely many) other "numbers" there. So far, nothing so peculiar. We already know such a conception of the continuum: infinitesimal or non-standard analysis. What is peculiar is what happens when we add extra numbers to $\mathbb{R}$; Peirce contends that, if we add enough numbers to $\mathbb{R}$, numbers are welded together and become one. That is how $\mathbb{R}$ becomes continuous. In his Peirce's Logic of Continuity, Fernando Zalamea suggests that Peirce's continuum can be nicely captured by category theory. However, Zalamea just gives us the outline and does not really show us how Peirce's continuum fits into the category-theoretic framework. In this paper, I will examine the relation between Peirce's and the category-theoretically constructed continua. For that purpose, first, I will briefly present Peirce's late conception of the continuum, and then, show how the continuum is constructed in category theory. Lastly, I will compare these two conceptions and draw some philosophical implications.

George Heine (Independent Scholar), **Mathématiques: Une Promenade Parisienne (A Math Walk in Paris)**

To both students and teachers, the history of mathematics offers rich opportunities for enhancing their understanding. Looking at the places connected with mathematical history brings about the possibility of a deeper awareness of how the development of mathematics may have been influenced by and brought influence to the larger culture.
No city in the world offers more mathematical history connections than Paris. Using sites of modern Paris as guideposts, we will explore some of these connections, and meet a few of the many mathematicians who lived, worked, and played in the City of Lights. As a part of the tour, we shall look at at least one or two unexpected and intriguing coincidences in space and time.
This talk may be of special interest to any who are planning to attend HPM 2016 in Montpellier, and considering a stopover in Paris.

Matthias Jenny (MIT), **The 'If' of Relative Computability**

I develop a theory of counterfactuals about relative computability, i.e. counterfactuals such as "If the validity problem were algorithmically decidable, then the halting problem would also be algorithmically decidable," which is true, and "If the validity problem were algorithmically decidable, then arithmetical truth would also be algorithmically decidable," which is false. These counterfactuals are counterpossibles, i.e. they have metaphysically impossible antecedents. They thus pose a challenge to the orthodoxy about counterfactuals, which would treat them as uniformly true. What's more, I argue that these counterpossibles don't just appear in the periphery of relative computability theory but instead they play an ineliminable role in the development of the theory. Finally, I present and discuss a model theory for these counterfactuals that is a straightforward extension of the familiar comparative similarity models.

Fabio Lampert, **Actually, Tableaux, and Two-Dimensional Modal Logic**

In this paper we present tableau methods for a two-dimensional modal logic called 2DML. Although models for such logics are well-known, proof systems remain rather underdeveloped. The fact that the logic contains doubly-indexed formulas motivated the construction of what we call "2D-tableaux," where the indices informally denote actual and possible worlds. This procedure is interesting for it can be easily generalized to cover a variety of two-dimensional logics. Furthermore, the majority of the axiomatizations of two-dimensional logics have been developed at the propositional level, whereas here we present sound and complete systems for both the propositional and first-order cases considering constant and variable domains, respectively. The system 2DML is a conservative extension of the modal logic S5, but it contains a new operator called "actually simpliciter."

Jean-Pierre Marquis (Université de Montréal, Montréal), **Foundations of Mathematics: A Science**

In this talk, I want to argue that the foundations of mathematics in the 19th and 20th centuries moved from being a collection of philosophical issues to a scientific discipline. This is an important shift with definite philosophical consequences which have not, I believe, been appreciated to their full extent and, even, have been forgotten recently by philosophers of mathematics. Of course, from an historical point of view, the development of the field is convoluted and far from being linear and homogeneous. I want to argue that nonetheless the end result is clearly that the discipline ought to be presented as being scientific and that this perspective changes the very nature of the field and its impact for philosophy of mathematics.

Connor Mayo-Wilson, Colin Marshall (University of Washington), **Descartes' *Géométrie* and Non-Propositional Meta-Theory**

Contemporary epistemology focuses on propositional knowledge and justification; contemporary mathematics and logic provide rigorous models of propositional deduction. In contrast, some modern philosophers, including Descartes and Locke, endorse non-propositional knowledge and deduction. This raises the question of what non-propositional meta-theory might look like. What is the analog of a valid (i.e., truth-preserving) inference for deductions involving non-propositional objects? What is the analog of completeness for non-propositional deductions?
In this paper, we argue that the permissible constructions in Descartes' *Géométrie* are (nomological) possibility-preserving, in the same way that valid inferences of propositional deductions are truth-preserving. In Cartesian terms, all constructible curves describe nomologically possible motions. We also argue that Descartes affirms a non-propositional "completeness" theorem: constructible curves include all knowable possible objects of geometric knowledge (where 'knowable' is 'knowable by finite beings'). Our work explains the broader philosophical significance of Descartes' distinction between geometrical and mechanical curves.

Paul McEldowney (University of Notre Dame), **Bolzano against Kant's Pure Intuition**

Throughout his writings, Bernard Bolzano fiercely criticized Kant's view that mathematical cognition proceeds from the construction of concepts in pure intuition—that is, the forms of sensibility, space and time.
This paper focuses on Bolzano's criticisms that (i) by its nature, intuition cannot account for the necessity or universality of mathematical cognition or inference ("The Categorical Objection"); and that (ii) even if intuition can help discover a mathematical proposition as being true, intuition is and has been shown to be dispensable in securing mathematical knowledge and inference ("The Indispensability Objection"). The purpose of this paper is to clarify (i) and (ii) from the point of view of Bolzano's overarching project

of reforming the sciences in his Wissenschaftslehre. Once this is done, this paper defends Kant against Bolzano's objections. This paper not only argues for the resilience of Kant's view in the face of (i) and (ii) but it aims to better understand Bolzano's own view of mathematical cognition by contending that both these objections stem from a rejection of the need to undergo an examination of the principles and limits of human cognition in order to secure and explain reliable forms of inference and claims to a priori knowledge.

Corey Mulvihill (University of Ottawa), **Proofs of intermediate logics from intuitionisitic logic plus epsilon and the ontological status of multivalent concepts**

When one adds Hilbert's choice operator epsilon to intuitionistic logic one can show that certain intuitionistically invalid principles are derivable. These results provide a finer-grained basis for Dummett's contention that a commitment to classically valid but intuitionistically invalid principles reflects metaphysical commitments, than can simply an analysis of the logical operators. Furthermore these results show that questions of realism and anti-realism are not an "all or nothing" matter, but that there are plausible metaphysical stances between the poles of anti-realism (corresponding to acceptance of only intutionistic logic) and realism (corresponding to acceptance of classical logic). Different sorts of ontological assumptions yield intermediate rather than classical logic and these positions between classical and intuitionistic logic link up in interesting ways with our intuitions about issues of objectivity and reality. They do so usefully by linking to questions around intriguing everyday concepts such as "is smart," which I suggest involve a number of distinct dimensions which might themselves be objective, but because of their multivalent structure are themselves intermediate between being objective and not.

Andrea Pedeferri (George Washington University), **The Situation of Logic in Italy from Peano to WWII**

In the first half of the last century, logical studies in Italy had been dominated by the figure of Giuseppe Peano, who deeply influenced many logicians worldwide (e.g. Bertrand Russell). The Italian Logic group headed by the Turin logician and mathematician established itself as one of the strongest and most innovative on the international scene. Unfortunately, the school born from this circle never did become a leading one; on the contrary, it died slowly. The aim of this paper is to identify and clarify the cultural, methodological and technical factors in the first half of 20th century which brought about the discontinuity of research in logic in Italy.

Jared Richards (University of Western Ontario), **Category Theory for the Philosophy of Mathematics**

The goal of this paper is to get philosophers of mathematics to take category theory seriously. To accomplish this goal I address two questions: (i) What is the general aim of philosophy? (ii) Does category theory help or enable us to accomplish this aim when particularized to the subject matter of mathematics?

I use the aim provided by Sellars (1962) to answer (i). To answer (ii), first, I particularize Sellars' aim to the subject matter of mathematics. The aim that results, notably, is kindred to that which a number of mathematicians (e.g., Eilenberg, Steenrod, MacLane, Lawvere, Caramello, and Chang) give (explicitly or implicitly) to the discipline of mathematical foundations. Speaking roughly, their aim is not the "justification" of mathematics; rather, it is a certain type of "deep" or "reflective" but still operational understanding of its entire subject matter. I then answer (ii) affirmatively. I argue that category theory gives us the conceptual means to accomplish the particularized aim in a desirable and philosophically fruitful manner. Furthermore, both traditional set theory and (homotopy) type theory, I argue, do not. Thus, as philosophers of mathematics, we should take category theory seriously because it gives us something that the usual foundational approaches to mathematics do not.

V. Frederick Rickey (West Point), **E228**

How's that for the shortest title ever? How can you decide if a number is the sum of two squares? Euler begins with the dumbest possible algorithm you can think of: Take the number, subtract a square, and check if the remainder is a square. If not, repeat, repeat, repeat. But Euler, being Euler, finds a way of converting all those subtractions into additions. Then he does several things to speed up the computation even more (but, sadly, does not explain himself very well). He applies this to 1,000,009, and — in less than a page — finds that there are two ways to express this as a sum of squares. Hence, by earlier work in E228, it is not a prime. Amusingly, when he later described how to prepare a table of primes "ad milionem et ultra" (E467), he includes this number as prime. As a consequence, he then feels obliged to write another paper, E699, using another refinement of his method, to show that 1,000,009 is not prime.

Dirk Schlimm (McGill University), **Frege's Begriffsschrift notation: Design principles and trade-offs**

Well over a century after its introduction, Frege's two-dimensional Begriffsschrift notation is still considered mainly a curiosity that stands out for its clumsiness rather than anything else. It stands out by its two-dimensional layout with symbols for logical relations (implication and negation) on the left and the propositional content on the right. I will introduce the propositional fragment of the notation and show its close connection to syntax trees, thereby arguing for the perspicuity and readability of the notation. Then I will present the aims that Frege pursued with his system together with his considerations regarding possible difficulties with the notation because of its unfamiliar look. In addition, Frege's justifications for the design principles underlying the Begriffsschrift are discussed, about which he was very explicit in his replies to early criticisms and unfavorable comparisons with Boole's notation for propositional logic. Despite the fact that this discussion is mainly about Begriffsschrift, it highlights some important trade-offs with regard to notations in general. In sum, my discussion reveals that, contrary to popular opinion, Begriffsschrift is in fact a well thought-out and carefully crafted notation that intentionally exploits the possibilities afforded by the two-dimensional medium of writing like none other.

Jonathan P. Seldin (Lethbridge University), **Some Philosophical Results on Incompleteness**

This talk, which continues a presentation to the CSHPM at Cambridge, England in 1204, will begin with a look at the proof of Gödel's First Incompleteness Theorem and related results (such as the undecidability of the halting problem). I will analyze enough of the proofs involved to show that the limitation has to do with our ability, either by ourselves or with the aid of computing devices we can build, to use rules to completely and correctly characterize such things as provability in formal systems or whether or not algorithms will halt for given inputs. The results are therefore not limitations on what rules can be written, but whether we humans can successfully use them for their intended purpose.
As is well known, these results can be proved only for strictly formalized situations, but they might hold of other proposed uses of rules, even if they cannot be proved for such cases. Again, the question is whether we humans have the ability to use these rules for their intended purpose(s). In informal situations in which we cannot prove incompleteness (or completeness, for that matter), we might ask what it might mean in practice if incompleteness is true.

Edward Shear, Jonathan Weisberg, Branden Fitelson (University of California Davis), **Two approaches to belief revision**

In this paper, we compare and contrast two methods for revising qualitative (viz., "full") beliefs. The first method is a nave Bayesian one, which operates via conditionalization (and, more generally, via mechanical/minimum distance updating) and the minimization of expected inaccuracy. The second method is the AGM approach to belief revision (which can also be understood in terms of mechanical/minimum distance updating). Our aim here is to provide the most straightforward explanation of the ways in which these two methods agree and disagree with each other, when it comes to imposing diachronic constraints on agents with deductively cogent beliefs. Some novel convergences and divergences between the two approaches are uncovered. Surprisingly, we establish that when deductively cogent agents revise by new information (i.e. a proposition previously not believed) the two approaches agree on the appropriate revision so long as a Bayesian's Lockean threshold is either no more than the inverse of the Golden Ratio or, as is well known, 1. Also surprisingly, the only divergences between the two involve violations of AGM's Vacuity axiom, which corresponds to non-monotonic logic's Rational Monotony.

Jeremy Shipley (Harper College), **Poincaré on the Foundation of Geometry in the Understanding**

In this paper I will offer an interpretation of Henri Poincaré's views on the foundations of geometry. I will present the received view and the textual evidence for it. I will argue that this view must be revised to properly account for Poincaré's emphasis on the role of algebraic groups as "forms of the understanding." According to the received view, Poincaré understood geometric axioms to be uninterpreted schemata that define mathematical concepts without reference to intuited objects. The view is supported by the natural interpretation it lends to Poincaré's statements on definitions and axioms, consistency, and conventionalism, as well as his use of reinterpretation/translation arguments to argue for the sensibility of geometry. Despite this support, the received view does not well account for Poincaré's repeated insistence that geometry is the study of a group. Against the received view, I argue that geometric terms are not, in Poincaré's considered view, defined implicitly by axiom schemata, but instead are given putatively nominal definitions as invariants of certain group actions. Surprisingly, axioms are, after all, interpreted in Poincaré's view. Support for the revised view is drawn from relevant texts reasons for accepting the received view are reconsidered.

Joel Silverberg (Roger Williams University), **Napier, Torporley, & Menelaus — A closer look at Augustus De Morgan's observations on early Seventeenth-century restructuring of planar and spherical trigonometry**

An effort to reorganize and systematize planar and spherical trigonometry began in the 15th-century with the work of Regiomontanus, extended throughout the 16th-century with work by Otto, Rheticus, Pitiscus, and Fincke, and continued into the 17th-century by Napier, Torporley, Viète, and others. During the 18th- and 19th-century, publications by Euler, Taylor, Fourier, and Gauss extended the role of trigonometric functions into new areas including power series, and complex functions of complex variables. An analysis of De Morgan's criticism of Napier's and Torporley's efforts in this area sheds light on the challenges to an historian of mathematics of one era attempting to understand the thought process of mathematicians living in earlier times. In particular, we focus on two areas: — the historian's knowledge of future mathematical developments and modes of expression unknown to those living in the earlier period, and secondly an incomplete, inaccurate, or absent knowledge on the part of the historian of definitions, references, or conventions well-known to those of the earlier era. These definitions, references, and conventions were often used without comment or explanation, and occasionally used without mention, since the writer could assume them to be common knowledge to the readers of his time, and that their use would be understood by his readers, even if that use was implicit.

James Tappenden (University of Michigan), **Frege, Carl Snell and Romanticism; Fruitful Con-**

cepts and the 'Organic/Mechanical' Distinction

A surprisingly neglected figure in Frege scholarship is the man Frege describes (with praise that is very rare for Frege) as his "revered teacher", the Jena physics and mathematics professor Carl Snell. It turns out that there is more of interest to say about Snell than can fit into one paper, so I'll restrict attention here to just this aspect of his thought: the role of the concept of "organic", and a contrast with "mechanical". Snell turns out to have been a philosophical Romantic, influenced by Schelling and Goethe, and Kant's Critique of Judgement. The paper also goes beyond Snell to explore other figures at Jena, particularly in the salon Snell sponsored and that Frege attended. Here too the the "organic/mechanical" contrast, understood in a distinctively Romantic fashion, had reached the status of "accepted, recognized cliché". More generally, Frege's environment was more saturated with what we now call "Continental philosophy" than we might expect. (Recently this "Continental" dimension of Frege's environment has been explored by Gottfried Gabriel and others, with an emphasis on neo-Kantianism and Herbart. This paper develops a different dimension: the speculative idealism informing German Romantic biology.)
The payoff of this context-setting for our reading of Frege's texts is this: many expressions and turns of phrase in Frege that have been regarded as vague, throwaway metaphors turn out to be literal references to theories that had been worked out in extensive detail by the people Frege spent time with day-to-day in his immediate Jena environment. This recognition allows us to see that many Fregean remarks were not disconnected, scattered asides, but reflect a connected picture of the nature of mathematical thought. In particular, this is true of Frege's account of "extending knowledge" via "fruitful concepts" and his rejection of the idea that logic and mathematics can be done "mechanically" (as with Jevons' logic machines, or Fischer's "aggregative mechanical thought"). Frege appeals to "organic connection" and speaks of fruitful concepts as containing conclusions "like a plant in its seeds". Frege would have expected his apparent metaphors to have been understood in a very specific way, as alluding to a recognized contrast between "organic" and "mechanical" connection, [mechanische/organische Verbindung and cognates] that was applied by Snell and those close to him not only to distinctions between biological and physical reasoning but also to distinctions of types of reasoning in arithmetic and geometry. Snell's account of conceptual development as well as his account of the development of species were structured around the idea of "development from a seed". In addition, Snell drew explicitly a connection that is only tacit in Frege, between the "organic" structure of fruitful concepts, and their fruitfulness - i.e. their potential for supporting novel insights. Snell's vision of the connection between organic structure and creativity draws on Kant's Critique of Judgement, which turns out to have been an unexpectedly salient touchstone in Frege's world.

Aaron Thomas-Bolduc (University of Calgary), **Between Logicism and Neo-Logicism**

The logicist program, conceived by G. Frege and redeveloped by B. Russell and A.N. Whitehead around the turn of the century, was a serious contender for the foundations of mathematics until mid-century. At that time, logicism was generally to be taken to be a continuation of program of the Principia rather than a direct continuation of Frege's program. In the 1980s however, C. Wright and R. Hale conceived a new program, more directly related to Frege's original conception. The threads coming out of those developments in the '80s have come to be known as neo-logicism. The gap between the decline of logicism and the rise of neo-logicism is the concern of this paper. More specifically, I will examine the positive contributions to abstractionist philosophy of mathematics that paved way for the development of neo-logicism in the '80s. I will begin with a discussion of C. Hempel's influential article from 1945, and pick out and discuss some major developments through the following four decades. Of particular interest are P. Benacerraf's PhD dissertation, M. Dummett's Frege: Philosophy of Language, and C. Parsons' and G. Boolos' early work on Fregean philosophy of mathematics and surrounding logical issues.

Sandra Visokolskis (National University of Cordoba), **Greek geometrical analysis and a plausible**

oriental source in the method of single false position: a discussion

Western classical historiographical tradition has tended to describe under the heading of "Mesopotamian Mathematics" to a set of pre-Islamic cultural practices —Sumerian, Babylonian, Assyrian, among others—, characterized pejoratively as a "child" origin of later Greek developments, as well as to Ancient Egypt Mathematics, thereby purporting to show an increasing continuity in a Western evolutionary process.
In the opposite direction, this paper attempts to analyze the known geometric method of analysis, released by Pappus of Alexandria in the fourth century. This method is considered of genuinely Greek origin—VI b C. century—, based on the first reports of the historiographical tradition of Mathematics, which extends to include the arguments of the nineteenth century colonialists. However, further research—Høyrup, Freiberg, Robson, Clagett, Rossi, Imhausen among others—poses precedents of Oriental strategies that would postulate close relationship with the Greek geometrical analysis, centuries before Western hegemony.
In this sense, this research focuses on its possible oriental background in the method of single false position and other types of false assumptions. The hypothesis is based on an alternative interpretation of the notion of analysis that respects the standard orientation. However, it is supplemented by adding certain elements which, presumably, are central to its description.

Richard Zach (University of Calgary), **The Decision Problem and the Model Theory of First-order Logic**

The emergence of first-order logic and its metatheory is commonly seen as a switch from a purely axiomatic development of the logical systems in the Hilbert school to a metalogical view that incorporates model-theoretic methods. Although one origin of these model theoretic methods undoubtedly can be found in the work of Skolem and later Gödel and Tarski, to whom they are usually credited, even in the Hilbert school itself work on the decision problem independently gave rise to model-theoretic thinking, occasioned by the needs for proving decidability. The episode shows how mathematical practice can force fundamental changes in methodology.