CSHPM SCHPM

The Canadian Society for History and Philosophy of Mathematics / La Société canadienne d'histoire et de philosophie des mathématiques

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The British Society for the History of Mathematics
La Société britannique de l'histoire des mathématiques

4th Joint Meeting / 4ieme Réunion mixte
Concordia University / Université Concordia
Henry F. Hall Building
1455 de Maisonneuve Blvd. W.
Montréal, Québec, Canada

FRIDAY, JULY 27, 2007

Room H-415

8:00 AM	**Coffee & bagels**
8:30 AM	**Welcome**

Parallel Session I A – Room H-415 Presider: Hardy Grant

9:00 AM	**Josipa G. Petrunic** In the wake of empiricism: British empiricist traditions in mathematical thought (1860-1880) and Felix Klein's Erlanger Program as local responses
9:30 AM	**Dirk Schlimm** On the creative role of axiomatics in the discovery of lattices
10:00 AM	**Elaine Landry** How To Be A Structuralist All The Way Down
10:30 AM	**Michel Serfati**. From Marshall Stone to Saunders Mac Lane. Elements for an epistemology of contemporary mathematics.
11:00 AM	**Jean-Pierre Marquis** The early history of categorical logic in Montreal

<u>Parallel Session I B – Room H-407</u> Presider: Sylvia Svitak

9:00 AM	**Janet Beery** Navigating Between Triangular Numbers and Trigonometric Tables: How Thomas Harriot Developed His Interpolation Formulas
9:30 AM	**Roger Godard** Some examples of Symmetry and Mathematics in the XVIIth, XVIIIth, XIXth Centuries
10:00 AM	**David Orenstein** The Archival Record of Education and Research in the Mathematical Sciences in Nouvelle France and Bas Canada
10:30 AM	**David Bellhouse** The Problem of Waldegrave
11:00 AM	**Nathan Sidoli** Ptolemy's *Planispaerium*: Reflections arising in editing the Arabic text. (Joint work with J. L. Berggren.)
11:30 AM	**Gavin Hitchcock** The many faces of "Analysis" in the making (1750-1850)

12:00 PM **LUNCH BREAK / EXEC MEETING**

<u>Parallel Session II A – Room H-415</u> Presider: Amy Ackerberg-Hastings

2:00 PM	**Bart Van Kerkhove** The historicity of mathematics: computer proof
2:30 PM	**Makmiller Pedroso** Realism and Mathematical Truth
3:00 PM	**Jason Douma** Philosophical Intelligences: a potential model for teaching mathematics
3:30 PM	**Miriam Lipschutz-Yevick** Poetic Metaphor and Mathematical Proof: A Shallow Analogy

<u>Parallel Session II B – Room H-407</u> Presider: Duncan Melville

2:00 PM	**Charlotte Simmons** Observations on Sir William Rowan Hamilton and George Boole
2:30 PM	**Sloan Despeaux** Mathematics sent Across the Channel: Nineteenth-Century British Mathematical Contributions to International Scientific Journals.
3:00 PM	**Joel Silverberg** "Circles of Illumination," "Parallels of Equal Altitude," and "le Calcul du Point Observé": Nineteenth Century Advances in Celestial Navigation
3:30 PM	**Israel Kleiner** Richard Dedekind (1831-1916): A path-breaking mathematician
4:00 PM	**Jean-Philippe Villeneuve** From Cauchy's integral to Lebesgue's integral axiomatization: When a new interpretation becomes a reinterpretation

4:30 PM **<u>Reception - LB Atrium</u>**
Library Building (directly across from the conference building)
1400 de Maisonneuve Blvd. W.

SATURDAY, JULY 28, 2007

Room H-415

8:30 AM **Coffee**

Parallel Session III A – Room H-415 Presider: Jean-Pierre Marquis

9:00 AM	**Duncan Melville** Fields and reciprocals: Some hints from Sargonic mathematics
9:30 AM	**Gregg De Young** QUṬB AL-DĪN AL-SHĪRĀZĪ'S "Demonstrations" of Euclid's Postulates: Mathematical and Metamathematics Issues
10:00 AM	**Glen van Brummelen** Telling Time in 10th-Century Baghdad: A New Instrument for Solar Timekeeping Comes to Light
10:30 AM	**Edward L. Cohen** Important Indian Calendars
11:00 AM	**Marina Vulis** Arabic contributions to cryptography
11:30 AM	**Munibur Chowdhury** T Vijayaraghavan (1898-1955) and A. Weil (1906-1998): A Tale of a Friendship

Parallel Session III B – Room H-411 Presider: Patricia Allaire

10:00 AM	**Jonathan P. Seldin** More Thoughts on the Teaching of Elementary Mathematics
10:30 AM	**Andrew Perry** The Advent of Conceptual Instruction in Nineteenth Century American Textbooks
11:00 AM	**George P. Styan** A Philatelic Introduction to Magic Squares
11:30 AM	**Hardy Grant** The Prehistory of "Experimental" Mathematics

12:00 PM	**LUNCH BREAK**
1:00 PM	**CSHPM ANNUAL MEETING – Room H-415**

Special Session --John Fauvel: In Memoriam – Room H-415 Presider: Jackie Stedall

2:15 PM	**Raymond Flood** John Fauvel: Life, Labours and Legacy
2:45 PM	**Snezana Lawarence** John's legacy: History of Mathematics in Mathematics Education

The Kenneth O. May Lecture – Room H-415

3:30 PM **C. Edward Sandifer** Five Pearls of Euler

Special Session—Leonhard Euler – Room H-415 Presider: Rob Bradley

4:30 PM	**Jordan Bell** Euler's summation of a divergent series involving the pentagonal numbers
5:00 PM	**Craig Fraser** Euler's Use of Divergent Series
5:30 PM	**Lawrence D'Antonio** How Euler Built the Britannia Bridge

SUNDAY, JULY 29, 2007

Room H-415

8:30 AM **Coffee**

Special Session—Leonhard Euler – Room H-415 Presider: Ed Sandifer

9:00 AM	**Rob Bradley** Euler's Resolution of Cramer's Paradox
9:30 AM	**Munibur Chowdhury** A Birthday Gift for Euler
10:00 AM	**Chris Baltus** Euler's Continued Fractions
10:30 AM	**Adrian Rice** What is the "birthday" of elliptic functions?
11:00 AM	**Amy Aackerberg-Hastings** Euler and the Enlightenment Mathematicians: A Scottish Perspective
11:30 AM	**Rüdiger Thiele** How did Euler change mathematics?

12:00 PM **LUNCH BREAK**

Special Session—Charles L. Dodgson – Room H-415 Presider: Fran Abeles

2:00 PM	**Tony Crilly** Being a Mathematics Undergraduate at Oxford and Cambridge in the Nineteenth Century.
2:30 PM	**Eugene Seneta** The "Inverse Probability" Controversy and Lewis Carroll. Read by Adrian Rice
3:00 PM	**George Englebretsen** The Dodo and the DO: Lewis Carroll and the Dictum de Omni.
3:30 PM	**Amirouche Moktefi** "My Logical Friends": Lewis Carroll and his contemporary logicians on the Barber shop problem.
4:00 PM	**Francine F. Abeles** The Tangled Tale of Dodgson's Condensation of Determinants

Abstracts – Alphabetical by Author

CSHPM/BSHM Joint Meeting

July 27-29, 2007

Francine F. Abeles
Kean University

The Tangled Tale of Dodgson's Condensation of Determinants.

Dodgson's condensation method has become a powerful tool in the automation of determinant evaluations currently. In this paper, I will describe the major steps on the "tangled" path beginning in the 20th century with its initial use in the study of the asymmetric signed matrix conjecture, including a combinatorial proof of it, and its role in the evaluation of a well-known 19th century determinant. I will then discuss additional developments that have led the way to its use in modern experimental mathematics.

Amy Ackerberg-Hastings
University of Maryland University College

Euler and the Enlightenment Mathematicians: A Scottish Perspective

Abstract: The professor of mathematics and natural philosophy at Edinburgh University, John Playfair (1748-1819) used expository writing, reviews, and historical accounts to shape British conceptions of mathematics and science. Specifically, he evaluated the contributions made to eighteenth-century mathematics by Leonhard Euler, Jean D'Alembert, Pierre-Simon Laplace, and other Continental mathematicians. His articles appeared in Transactions of the Royal Society of Edinburgh, the Edinburgh Review, and Encyclopaedia Britannica. The talk will explore the portraits Playfair developed in these writings and consider his body of work's own merit as propaganda and as primary source material for the history of mathematics.

Chris Baltus
State University of New York, Oswego

Euler's Continued Fractions

Abstract: When Euler first worked with continued fractions, by 1730, the subject consisted of a few formulas, largely from Wallis, and a few particular continued fractions. Euler established ties to differential equations and infinite series, and studied a variety of special forms. When he finished, continued fractions constituted a field within mathematics. His continued fraction work illustrates, or, better, exemplifies, his general approach: the brilliant exploitations of examples to arrive at general forms, the intense interest in computation, the discovery of connections between apparently distant ideas.

We will also see that his lesser interest in theory limited his achievement in the case of the Pell Equation, where the young Lagrange quickly surpassed him.

Janet Beery
University of Redlands

Navigating Between Triangular Numbers and Trigonometric Tables: How Thomas Harriot Developed His Interpolation Formulas

By 1611, Thomas Harriot (1560-1621) was developing finite difference interpolation methods, work that culminated in 1618 or later in his unpublished treatise, De numeris triangularibus et inde de progressionibus arithmeticis: Magisteria magna, in which he derived symbolic interpolation formulas and showed how to use them to interpolate in tables. This treatise and its influence have been the subject of recent research by the author and Jacqueline Stedall. The interpolation formulas that appear in Harriot's manuscripts vary in notation, structure, and method of application. In the present paper, we use these largely undated manuscripts to show that Harriot probably discovered his interpolation formulas independently of past and contemporary mathematicians and to show how he may have developed and refined his methods over time.

Jordan Bell
Carleton University

Euler's summation of a divergent series involving the pentagonal numbers

Abstract: Euler's pentagonal number theorem gives the series expansion of the infinite product $(1-x)(1- x^2)(1- x^3) \dots$. It is called the pentagonal number theorem because the exponents in the series expansion are the pentagonal numbers $n(3n \pm 1)/2$. The pentagonal number theorem was used by Euler to prove recurrence relations for the partition and sum of divisors functions. In "De mirabilibus proprietatibus numerorum pentagonalium" (E542), Euler uses the pentagonal number theorem to sum a divergent series involving the pentagonal numbers. I will explain this argument, and also discuss several other of Euler's uses of infinite products to sum series.

David Bellhouse
University of Western Ontario

The Problem of Waldegrave

Pierre Rémond de Montmort, in his 1713 book *Essay d'analyse sur les jeux de hazard*, mentions a M. de Waldegrave, a man who was corresponding with him on probability problems. This Waldegrave gave the first expression of a mixed strategy solution in game theory and lends his name to Waldegrave's Problem in probability. Who this Waldegrave was has never been properly determined. Three authors have ventured to identify the man; all three were close, but incorrect. If the Internet is anything to go by, the most popular incorrect choice is due to Harold Kuhn, writing in the preface to *Precursors in*

Mathematical Economics: An Anthology by Baumol and Goldfeld published in 1968. His choice is the English aristocrat James Waldegrave, 1st Earl Waldegrave. This Waldegrave was initially educated in France and later served as the British ambassador to Paris and Vienna. The correct Waldegrave turns out to be the first earl's uncle, Charles Waldegrave, an active Jacobite. A very sketchy biography of Charles Waldegrave is provided and his contributions to probability are reviewed.

Rob Bradley
Adelphi University

Euler's Resolution of Cramer's Paradox

Abstract: In a September 1744 letter, Gabriel Cramer introduced Leonhard Euler to a problem in the theory of cubic curves, the generalization of which has become known as Cramer's Paradox. In a 1750 paper (E147), Euler eventually proposed a resolution of Cramer's Paradox by introducing a notion related to linear independence. In Euler's October 1744 reply to Cramer's letter, which has only recently come to light, he correctly identifies the direction in which the paradox ought to be resolved, arguing by analogy in the case of conic sections. In this paper, we will examine Euler's arguments in both the 1744 letter and the 1750 article.

Munibur Rahman Chowdhury
University of Bangladesh

T Vijayaraghavan (1898-1955) and A. Weil (1906-1998): A Tale of a Friendship

Vijayaraghavan is remembered for his work in number theory (especially Diophantine approximation) and in analysis, while Weil was one of the towering figures of 20th century mathematics. Their life-long friendship began in 1930 when Weil, serving a two-year stint as head of the Department of Mathematics of Aligarh Muslim University, India, appointed Vijayaraghavan as a Lecturer. Next summer, Vijayaraghavan joined the University of Dhaka (former spelling: Dacca) as a Reader. Weil visited Dhaka as a personal guest of Vijayaraghavan on his way back to France in early 1932. This visit, lasting some weeks, was mathematically fruitful. They met again at the International Congress of Mathematicians in 1936 at Oslo. They met for the last time in 1951, when Weil had Vijayaraghavan invited to the University of Chicago for one quarter.

While an undergraduate at Presidency College, Madras, Vijayaraghavan showed such talent that he was typed as a second Ramanujan was sent to Oxford to study with G.H. Hardy, Savilian Professor of Geometry at Oxford during 1919-1929. In a 1984 letter to the author, Weil sums up his impression of Vijayaraghavan as a mathematician as follows: "no doubt, Vij was a mathematician of great penetration; unfortunately, he had been too much under the influence of Hardy at the outset of his career to have learnt to broaden his outlook."

Munibur Rahman Chowdhury
University of Bangladesh

A Birthday Gift for Euler

Abstract: We give an account of Euler's seminal contribution to the theory of residues (1761) culminating with Euler's theorem $a^{\varphi(n)} \equiv 1$ (mod n) for very integer a coprime to n, and Euler's formula $\varphi(n) = n \prod_{p|n} (1-1/p)$. This work of Euler's is one the sources of group theory. Here we use group theory to prove these results. In this exposition, we assume only a budding acquaintance with the group concept. Everything else is developed *ab initio*; although a rudimentary knowledge of elementary number theory would be an advantage.

On the occasion of the tercentenary of Euler's birth, we propose that the multiplicative group of the prime residue classes modulo n be called the Euler group modulo n, and be denoted by E_n.

Edward L. Cohen

Important Indian Calendars

India is a large country and it was around for many years. Thus it had many calendars (at least 30) based on astrological and astronomical methods. We examine the more important calendars, which include the Vedic, Hindu, and the modern day ones.

Tony Crilly
Middlesex University

Being a Mathematics Undergraduate at Oxford and Cambridge in the Nineteenth Century.

Cambridge University was the centre for mathematics in England during the nineteenth century, and a mathematics education at Cambridge makes a stark comparison with the mathematical education a student would receive at Oxford where Charles Dodgson was a college lecturer. An outline of curriculum at both places will be given and the student experience will be compared.

Lawrence D'Antonio
Ramapo College

How Euler Built the Britannia Bridge

Abstract: It is a remarkable but little known fact that Leonhard Euler built the Britannia Bridge connecting Wales and the isle of Anglesey. The bridge, considered a marvel of engineering for its time, was constructed in 1850. This talk will consist primarily in explicating the contradictions contained in the previous sentences.

Gregg De Young
The American University in Cairo

Qu□b Al-Dīn Al-Shīrāzī's "Demonstrations" Of Euclid's Postulates: Mathematical and Metamathematical Issues

Qu□b al-Dīn al-Shīrāzī (634/1236 – 710/1312) is credited with the first rendition of Euclid into Persian (completed 698/1298). This translation is not based on the primary Arabic transmission of Euclid, but on the *Ta□rīr* of Na□īr al-Dīn al-□ūsī (597/1201 – 674/1272). The translation is typically quite literal, but displays some surprising features, such as the removal of nearly all of al-□ūsī's alternate demonstrations (borrowed mainly from Ibn al-Haytham's *Shukūk Kitāb Uqlīdis*). Also noteworthy is the introduction of "demonstrations" for the postulates of Book I. Many of these "demonstrations" have a long history – similar discussions can be seen already in the commentary of Proclus. The "demonstration" for the parallel lines postulate of Euclid, however, is unique, so far as I can determine, to al-Shīrāzī. In the translation, it replaces the better-known "demonstration" of al-□ūsī. This "demonstration" will be the focus for my presentation.

Apart from the internal mathematics of the "demonstration," the translator's manipulation of the text raises meta-mathematical issues: (1) what freedom did medieval translators have when translating mathematical material? What sorts of interference was the translator permitted to make in the text when translating? And (2) why was the Persian transmission based on al-□ūsī and not on the primary Arabic transmission of the *Elements*? The paper will conclude with some speculative suggestions for possible answers to these questions.

Sloan Despeaux
Western Carolina University

Mathematics sent Across the Channel: Nineteenth-Century British Mathematical Contributions to International Scientific Journals

This talk will consider the range of British participation in the international mathematical publication community during the nineteenth century through an analysis of British mathematical contributions to scientific journals outside of Britain. The number and types of papers presented by British mathematicians to these journals characterize the role of foreign publication in nineteenth-century British mathematics. Moreover, the isolation of educational, societal, and personal circumstances, which motivated British mathematicians to present their work to foreign journals highlights limited but concentrated groups of mathematicians committed to developing and strengthening international mathematical ties with Britain.

Jason Douma
University of Sioux Falls

Philosophical Intelligences: a potential model for teaching mathematics

Intellectual wrangling over the ontological and epistemological foundations of mathematics is conducted largely, and understandably, outside the more pragmatic environment of the mathematics classroom. But might the claims of the various philosophical positions (formalism, structuralism, humanism, etc.) provide insights into how mathematics should be taught and learned? Even if the ultimate philosophical questions remain unresolved, are there immediate pedagogical benefits to be gained from the very presence of multiple philosophical perspectives? This talk will address these questions by considering the implications of "teaching to multiple mathematical epistemologies," in much the same spirit as the broader educational call to teach to Gardner's multiple intelligences. Much of the material for this talk will be drawn from lessons learned in teaching an undergraduate philosophy of mathematics class, as well as an introductory liberal arts mathematics class.

George Englebretsen
Bishop's University

The Dodo and the DO: Lewis Carroll and the Dictum de Omni.

There is not much new in Lewis Carroll's system of logic as presented in *Symbolic Logic* (1896/1977).It's the traditional term logic (syllogistic) initiated by Aristotle, with many accretions from Scholastic logicians to the nineteenth century algebraists. What is new in his book is the large number of original technical methods he devised to make the learning and application of that logic more mechanical and easier. A (for many, such as Leibniz, *the*) key rule of syllogistic reckoning is what was traditionally known as the Di*ctum de Omni (et Nullo), DO.* The algebraic logicians, by extending logic beyond simple 2-premise syllogisms, confronted what was called the "elimination problem." Carroll offered a solution to it. As well, he followed traditional logicians in his acceptance of DO as a fundamental rule. Elimination and DO are intimately related. My aim is to show how Carroll's version of syllogistic logic can be seen as a precursor to a later revitalized term logic; one which takes term elimination to be a matter of substitution; and substitution is what DO is all about.

Raymond Flood
Kellogg College, Oxford University

John Fauvel: Life, Labours and Legacy

John Grant Fauvel was born on 21st July 1947 in Glasgow. This year would have seen his 60th birthday where it not for his untimely death at the age of 53. He was an energetic scholar, teacher and historian of mathematics and of particular importance to him was the use of the history of mathematics in education. As President of the BSHM

and subsequently as Newsletter editor he fostered international collaboration and discussion. I am delighted to have the opportunity at this joint meeting between the British and Canadian societies to share with you some thoughts on John's life, labours and legacy.

Craig Fraser
University of Toronto

Euler's Use of Divergent Series

Abstract: The paper examines some of Euler's papers on divergent series, situating his analysis and understanding of the subject with respect to the outlook of late nineteenth-century theory of summability.

Roger Godard
Royal Military College of Canada

Some examples of Symmetry and Mathematics in the XVIIth, XVIIIth, XIXth Centuries

In 1952, Hermann Weyl, a notorious Mathematician, wrote a book where he stated explicitly that “from the vague notion of symmetry, of harmony of proportions… We develop gradually the geometrical concept of symmetry under its different forms (bilateral symmetry, translation, rotation, ornamental symmetry and crystallography), to end up with the invariance of a configuration of elements…” These few sentence, have inspired this present work in choosing selected examples of Symmetry and Pure Mathematics. These examples are taken from the XVIIth, XVIIIth and XIXth centuries: Pascal’s triangle and the development of combinatorics and the binomial theorem; laws of probability (uniform, normal); the bilateral symmetry with cosine Fourier series (Euler, Clairaut, Lagrange, Gauss, Fourier), the Fourier definition of parity of a function; the rotational symmetry with the strength of columns (Euler, Lagrange), the translational symmetry with the Euler advection equation; the classification of crystals (Haüy), the emergence of the concept of commutativity, and finally the destruction of symmetry with direct sums or convolutions.

Hardy Grant
York University

The Prehistory of "Experimental" Mathematics

Now a flourishing branch of our discipline, with its own journal since 1992, "experimental" mathematics has been characterized as "the utilization of modern computer technology as an active tool in mathematical research". Taken literally, this definition obviously confines the history of the subject to the past few decades. But, no less obviously, the psychology and methodology implied by the definition have much older roots, some of which I shall here try to point out. In particular, one can ask to what

extent Euler may be placed among the pioneers, and the answer is that he could serve as "poster boy".

Gavin Hitchcock
University of Zimbabwe

The many faces of “Analysis"” in the making (1750-1850)

I will discuss some insights (and surprises) arising in the course of research for the writing of an introductory text on ``Mathematical Analysis with the Help of its History. This will include some (perhaps neglected) aspects of the work of Babbage and Peacock, and the later Cambridge Mathematics Journal contributors. Some contrasts will be discussed between Continental and British approaches and attitudes: Lagrange and Playfair, the Ecole Polytechnique and the Cambridge Analytic Society, Cauchy and De Morgan.

Israel Kleiner
York University

Richard Dedekind (1831-1916): A path-breaking mathematician

I will discuss Dedekind's mathematical work, focusing on his path-breaking contributions, including the founding of algebraic number theory, the definition of the real numbers in terms of "Dedekind cuts", the definition of the natural numbers in terms of sets, and the pointing (with Weber) to an analogy between algebraic geometry and algebraic number theory.

Elaine Landry
University of Calgary

How to Be a Structuralist all the Way down

This paper will consider the nature and role of axioms and use this to reconsider the current debates about the status of category theory and the algebraic approach to mathematical structuralism. I will first investigate the Frege-Hilbert debate with the aim of distinguishing between axioms as statements that are used to express or assert truths about a subject matter and an axiom system as schema that is used to provide a system of conditions for what might be called a relational structure. (Bernays [1967], p. 497) I will use this inquiry to reevaluate arguments against using category theory to frame a structuralist philosophy of mathematics. For example, Hellman has argued that category theory cannot stand on its own as a foundation for a structuralist interpretation of mathematics because the "problem of the home address remains" (Hellman [2003], p. 8 & 15). That is, because the axioms for a category merely tell us what it is to be a structure of a certain kind and thus its axioms are not assertory (Ibid., p. 7), we need a background theory which has axioms that are assertory, i.e., that assert the possible existence of systems so structured.

With aims similar to mine but with a decidedly different conclusion, Shapiro [2005] has claimed that the Frege-Hilbert debate can be used to show that the current structuralism debates are concerned with questions that consider the status of meta-mathematical axioms (as opposed to the mathematical axioms). That is, even if we agree with the Hilbert-inspired algebraic structuralist that any given branch is about any system that satisfies its axioms (Shapiro [2005], p. 74), to give criterion (of coherence, of consistency, of satisfiability) for the existence of such systems, we still need a background meta-mathematical theory which, itself, is assertory and so we cannot be algebraic structuralists all the way down. Our only other option, as proposed by Awodey [2004] is to kick way the foundational ladder altogether, and take the meta-mathematical set-theory, structure theory, or whatever, itself to be an algebraic theory (Ibid., p. 74). This option, however, is presented by Shapiro as a way not to be looked into because it has the unwanted consequence that mathematical logic is similarly liberated from theories our theorist can hold that satisfiability, consistency, or coherence implies existence, but she cannot maintain that any of these notions are mathematical matters (Ibid., p. 75) so that meta-mathematical matters are turned into non-mathematical, or 'philosophical', matters. My aim will be to show that category theory has as much to say about an algebraic consideration of meta-mathematical or logical structure as it does about mathematical structure and this without turning mathematical issues into 'philosophical' ones. Thus, we can use category theory to frame an interpretation of mathematical structuralism according to which we can be algebraic structuralists all the way down.

Snezana Lawrence
Simon Langton Grammar School for Boys, Canterbury

John’s legacy history of mathematics in mathematics education

This talk will focus on the influence John Fauvel had not only on me, but I believe, great number of mathematics teachers throughout the world. I will reminisce on the principles that John set in using the history of mathematics in maths education, and will share some of my memories of meeting John upon my first visit to the Open University in Milton Keynes, UK, in 1994. I hope that by examining not only John's influential trail in the mathematics education community in UK, I will also be able to contribute with some anecdotes from the time when I had a great pleasure to know John an instant, and abundant, source of inspiration, amazement and amusement (of the highest level!).

Miriam Lipschutz-Yevick
Rutgers,The State University

Poetic Metaphor and Mathematical Proof: a Shallow Analogy

In a recent article, Mathem*atical Analogy and Metaphoric Insight”* by Jan Zwicky (1, 2006) the author investigates the correspondences between the notion of metaphor primarily as it is used in poetry and that of analogy as it appears in the development of mathematical demonstrations. Although she clearly states that metaphors and mathematical analogies are not the same thing, she maintains that there are fundamental similarities, which suggest that both metaphors and mathematical analogies are species of "analogical reasoning." Analogy, the drawing on associations, is all-pervasive in our thinking and our language as well as our creative endeavors, be they artistic, scholarly or day-to-day. Zwicky argues for the kinship of metaphor and mathematical demonstration because in both cases the creation of new insights derives from discoveries of unsuspected analogies between facts long known but wrongly believed to be strangers to each other. Once again this kinship extends to all creative activity. I maintain that it is far-fetched to use this broad notion of analogy to project a tight embrace between two highly distinct domains of creative endeavor: poetic metaphor and mathematical proof. The first is supported mainly by an associative mode of thought; the second by rational deduction; the first will embellish – in its literal sense – the evoked analogies; the second will strip them down to their analytical content.

However, going beyond analogy, the investigation of the divergence in the further use and unfolding of analogy in these two domains points towards the expression of two *complementary* modes of thought, two "Languages of the Brain (2, 1971), (3,1975). The first of these is modeled by the digital logic of neural networks, the other, as I will illustrate, is mimicked by holographic pattern recognition. It is this dichotomy, which transcends Zwicky's claim of analogy as the salient attribute of metaphoric and mathematical creation. Even though both use analogy differently, their symbiosis may suggest a more insightful mode of thought.

Jean-Pierre Marquis
Université de Montréal

The early history of categorical logic in Montreal

Some of the key notions of categorical logic, e.g. regular categories, coherent categories, generic model, classifying topos, geometric logic, were formulated by category theorists in Montreal in the early seventies, primarily André Joyal and Gonzalo Reyes. In this talk, we will look at the situation one finds in Montreal in the sixties, where category theory and algebraic logic meet and interact in a unique manner, paving the way to the foregoing notions.

Duncan Melville
St. Lawrence University

Fields and reciprocals: Some hints from Sargonic mathematics

One of the characteristic features of Old Babylonian (c. 2000 – 1600 BC) mathematics is the way that division is performed as multiplication by the reciprocal'. In this talk, we present some hints from Sargonic (c. 2350 - 2200 BC) field measurement exercised that suggest a possible origin of the technique. No previous experience of Sargonic mathematics is necessary.

Amirouche Moktefi
Université Louis Pasteur

"My Logical Friends": Lewis Carroll and his contemporary logicians on the Barber shop problem.

Lewis Carroll's fame today as a logician is partly due to his "Achilles and the Tortoise" dialogue, published in the journal of philosophy *Mind* (April 1895). However, while this text attracted logicians only years after its first publication, it is another much less well-known *Mind* paper which became the subject of immediate controversy, and made Lewis Carroll known among his contemporary logicians. When the Barbershop problem appeared in July 1894, it was already the subject of dispute among British logicians. Lewis Carroll wrote numerous versions of the problem (eight were published by Bartley III in 1977), sent copies of them to the main logicians of the time and compared their solutions. In my presentation, using both published and unpublished material, I will discuss the genesis of the problem, the evolution of the debate, and make some statements on Lewis Carroll's relationship with his contemporary logicians.

David Orenstein
University of Toronto

The Archival Record of Education and Research in the Mathematical Sciences in Nouvelle France and Bas Canada

The Seminaire de Quebec in Quebec City (which also took over the education role of the College des Jesuites after the 1759 conquest) and the Seminaire des Sulpiciens in Montreal were the foundation stones of secondary and higher education in French Canada. Their archives (manuscripts, textbooks, and professional) collections are maintained respectively at the Musee de la civilisation in Quebec City and the Bibliotheque nationale in Montreal. This paper presents the results of exploring these collections in presenting the history of the mathematical sciences (mathematics, astronomy, physics) in Nouvelle France and Bas Canada (c. 1620 - 1840).

Makmiller Pedroso
University of Calgary

Realism and Mathematical Truth

In his 'Mathematical Truth', Benacerraf identifies two ways in which the discussion about mathematical truth may come up: (1) in presenting a semantic theory to mathematical propositions, and (2) in giving an account of mathematical knowledge. A semantics for mathematics can be formulated if we suppose that mathematical objects exist and, through the notion of satisfaction, define what is mathematical truth. However, as Benacerraf argues, this maneuver implies that mathematical knowledge is impossible because abstract entities like mathematical objects are causally inert. This argument relies on the premise that a semantic account of mathematics does not have to suppose that mathematical knowledge is attainable. My goal is to argue that this premise is false. More precisely, I claim that if the supposition that mathematical objects exist allows us to give a semantics for mathematics through the notion of satisfaction, then mathematical knowledge has to be a possible achievement. Therefore, the view according to which the semantics of mathematics is in conflict with the epistemology of mathematics is false.

Andrew Perry
Springfield College

The Advent of Conceptual Instruction in Nineteenth Century American Textbooks"

The first half of the Nineteenth Century represented a turning point of sorts for elementary mathematics textbooks used in the United States. Many books of this era make a serious effort to teach conceptual understanding of the mathematics underlying the computational algorithms they teach. They offer a balanced approach like contemporary textbooks, in contrast with the traditional style of demanding slavish memorization from the pupils. We will consider some feature of these newer conceptual textbooks and analyze their rise to prominence.

Josipa G. Petrunić
University of Edinburgh

In the wake of empiricism: British empiricist traditions in mathematical thought (1860-1880) and Felix Klein's Erlanger Program as local responses

The influence exerted by British empiricists in the reform movement at the University of Cambridge in the 1860s and 1870s has been an important topic for discussion in much recent scholarly literature. In this paper I will attempt to add to the discussion by highlighting the empirical philosophy of the mathematician William Kingdon Clifford (1845-1879), especially as it relates to his construction of bi-quaternions as measuring tools in non-Euclidean space. Parallel to Clifford's own work, Felix Klein (1849-1925) was developing his Erlanger Program, in which mathematicians could endeavour to study the properties of space that are invariant under a given group of transformations. I will argue that we can understand Klein's motivations in developing this program, in part, by reflecting upon the unique epistemological claims that British mathematicians such as Clifford were making in light of the English reception of non-Euclidean geometries. I will argue that Clifford and Klein were both engaged in a more general discussion regarding the nature and origin of mathematical knowledge, as well as the limits of mathematical *knowing.* I will compare both mathematicians' respective responses to the question of how non-Euclidean geometries altered the foundations of mathematics to highlight the way in which mathematical innovation is highly dependent upon local circumstances and intellectual cultures. In this case, we will see two mathematicians manoeuvring in different intellectual cultures, producing two very different interpretations of *meaning* as they struggle to come to terms with the geometrical works of Bernhard Riemann and Nikolai Lobachevskii.

Adrian Rice
Randolph-Macon College

What is the "birthday" of elliptic functions?

On December 23, 1751, Euler received a copy of a paper by Count Giulio Carlo de' Toschi di Fagnano on the lemniscate, which directly inspired the creation of Euler's general addition theorems for elliptic integrals. After his major contributions to the subject and the subsequent development and systematization of the theory by Legendre, elliptic functions became one of the dominant areas of mathematical research during the 19th century, leading Jacobi to call December 23, 1751 "the birth day of the theory of elliptic functions." But to what extent can the subject be said to have been born with Euler in 1751? After all, several other mathematicians, including Jacobi himself, are often credited with laying the foundations of what was to become the theory of elliptic functions, in which case its "birthday" could be anywhere from 1694 to 1829. By looking at the contributions of Euler, together with those of four other mathematicians, this talk will examine whether the theory of elliptic functions really did begin in 1751, or whether there is another date that could more accurately be described as "the birth day of the theory of elliptic functions."

C. Edward Sandifer
Western Connecticut University

The Kenneth O. May Lecture: Five Pearls of Euler

We look at five of Euler's best known and most beautiful mathematical results, the Basel problem, the polyhedral formula, the Euler identity, the Königsberg bridge problem and the Euler product formula. On the one hand, all are beautiful results, products of a creative genius of the highest order. On the other hand, Euler's presentations of each of these results has some sort of flaw that, on close examination, might make the modern reader uncomfortable. We consider this discomfort, reflect that what is a pearl to us is a great irritation to the oyster, and consider the nature of portioning out credit for mathematical discovery.

Dirk Schlimm
McGill University

On the creative role of axiomatics in the discovery of lattices

It is very common to find accounts of the use of axiomatics in science and mathematics that begin with a specific set of objects or a certain domain of being(s), say D, which an axiomatic system, say S, is intended to describe and characterize. Understood in this way, axiomatization is the process of finding an adequate S for a given D.

However, Aristotle's brief remarks about the introduction of a new notion, for what numbers, lines, solids, and times have in common based on the similarity of certain proofs about numbers, lines, solids, and times (*Analytica Posteriora*, *Bk. I, Ch. 5*) suggests the following procedure: (1) Take some domains D_1, D_2, D_3, etc. (2) Determine the corresponding systems S_1, S_2, S_3, etc. (3) Compare these systems and find a (sub-)system S' that is common to them. (4) Introduce a *new* notion D' as the domain of being for S'. Aristotle noticed that a scientific system S' can be used in this way to suggest new notions, objects, or domains. Thus, axiomatization is not necessarily a one-way process from D to S, but it can also lead one from S' to D'. This insight presupposes neither the notion of *formal* system, nor the possibility of multiple interpretations (although the latter would most likely be our way of expressing it). Since the domain D' is more *abstract* (in the sense of having only a subset of the properties) than the domains D_1, D_2, D_3, etc., the natural setting for such introductions of new notions is mathematics, since its objects are inherently abstract. Indeed, the mathematical notion of magnitude was introduced to express what the domains discussed by Aristotle have in common.

With a conception of formal systems at hand, by which I mean systems that can be interpreted in different ways, and which emerged in the 19th century, a second, related way of introducing new domains became possible: Only certain aspects of a single domain D are axiomatized by a system S, and then a new domain D', is introduced that is completely determined by S. As a result, this new domain is more abstract than D itself.

Furthermore, an axiomatic system S does not need to originate from a given domain D, but it can also be obtained through manipulation from another system of axioms. For example, the first axiom systems for non-Euclidean geometry were obtained in this way from given systems of Euclidean geometry. Only after their consistency was established, the new sets of objects, namely non-Euclidean points and lines, were introduced. This is a *third* way of introducing new domains.

In this paper I will present and discuss how the notion of *lattice* has been introduced independently by Dedekind, Schröder, and Birkhoff as examples for the three methods for introducing new domains of being based on axiomatic systems mentioned above, and I conclude that the axiomatic method is not only a way of systematizing our knowledge of specific domains, but can also be used as a fruitful tool for discovering and introducing new domains of being. Looked at it from this perspective and taking into account the role of axiomatics in modern mathematical practice, the creative aspect of axiomatics is brought to the fore.

Jonathan P. Seldin
University of Lethbridge

More Thoughts on the Teaching of Elementary Mathematics

Last year at CSHPM, I presented some ideas on the teaching of elementary mathematics that were partly inspired by the talk given by Keith Devlin at the summer 2005 meeting of the CMS (at which CSHPM also met), ideas involving the use of the history of mathematics to approach the foundations of analysis and the notion of proof in mathematics. In this paper, I will report on an attempt I made in the fall of 2006 to apply these ideas to a third-year course in analysis and what I see as the lessons of that attempt.

Eugene Seneta
University of Sydney

The “Inverse Probability” Controversy and Lewis Carroll (Read by Adrian Rice)

In the controversial application of inverse probability reasoning, the prior probability of a hypothesis or cause is modified on the basis of an experimental observation to a posterior probability. A contentious issue is the probabilistic expression of prior belief, especially of prior ignorance, which impinges on the nature of probability. The time from 1876, the expressed beginning of his interest in this issue, to 1893, when his "Pillow Problems" appeared, coincided with a controversy on the nature of probability between the logicians/frequentists (Venn, Chrystal) and probabilist Whitworth, who followed the more "Bayesian" footsteps of De Morgan and his mentor Laplace, and of Todhunter. The probability problems in "Pillow Problems" reflect Lewis Carroll's position within this context. The controversy lives on, albeit in more illuminated forms, within modern mathematical statistics.

Michel Serfati
Université Paris

From Marshall Stone to Saunders Mac Lane: Elements for an epistemology of contemporary mathematics

The word *ontemporary* when applied to mathematics, must be understood here opposite to *modern*. By modern, I mean early XXth mathematics, (roughly) up to Emmy Noether and Hilbert, the most influential treatise of which being doubtlessly Van der Waerden’s Modern Algebra. On the contrary, by contemporary mathematics, I mean here some of these important mathematical ideas which, on the one hand, arose after World War II, and on the other hand and principally, are to-day still in use and fruitful. Along these lines, a lot of Marshall Stone’s results in the 1930?s were a decisive turning point between the two periods, an early breakthrough from which arose completely new methods and mathematical conceptions, between algebra, geometry and topology. I shall examine below some conclusive Stone’s methods. Finally, these *elements for an epistemology* will appear at the end of the present conference essentially devoted to Stone and MacLane, as four main figures of thought, namely *ideal*, *categorical*, *duality* and *adjointness*.

Nathan Sidoli
University of Toronto

Ptolemy's *Planispaerium*: Reflections arising in editing the Arabic text. (Joint work with J. L. Berggren.)

The *Planispaerium* is a difficult work. It is written for a reader who has already mastered books I and II of the *Almagest*, has a thorough grounding in ancient spherical geometry and some understanding of the method of representing a sphere on the plane that we would call 'stereographic projection..' Because the requisite background knowledge is never made explicit, modern readers have developed a number of different opinions about Ptolemy's aims and his success in achieving these. In the process of editing and translating the medieval Arabic version of this lost Greek treatise, we have developed a new reading of the text that encompasses the project of the entire treatise and explains those features of the work that have struck modern readers as most obscure. In this talk, I will describe the foundations of this reading and sketch some of the issues it resolves.

Joel Silverberg
Roger Williams University

"Circles of Illumination," "Parallels of Equal Altitude," and "le Calcul du Point Observé": Nineteenth Century Advances in Celestial Navigation

By the early nineteenth century, the dream of determining the latitude and longitude of a vessel at sea had become a reality. The procedure was a lengthy one, first determining the latitude through observation of the sun at noon, and later by determining the longitude through observations taken when the sun bore, as nearly as possible, East or West. During this interval the position of the ship may have changed by hundreds of miles. Earlier estimates of latitude were no longer valid, yet the later calculations depended upon them. A dead reckoning was used to update the latitude to the time of the later observations, but this was an inherently inaccurate process. We explore the mathematical and geometric insights of two mariners, a Yankee sea captain and a French naval officer, whose discoveries overcame these and other problems and changed the face of celestial navigation.

Charlotte Simmons
University of Central Oklahoma

Observations on Sir William Rowan Hamilton and George Boole

Sir William Rowan Hamilton and George Boole are regarded as two of the greatest nineteenth century algebraists, and rightly so, as their work helped lay the foundations for abstract algebra. They are of significance to the history of science for more than just their mathematical contributions, however, as a study of their lives clearly demonstrates. For instance, Boole is a wonderful example of what can be accomplished through diligence and perseverance. Though economically disadvantaged, he surmounted all obstacles to become a successful mathematician. Though Hamilton did not have the same financial difficulties as Boole, he had many obstacles to overcome in his personal life. While the two came from very different backgrounds, Boole and Hamilton actually shared many philosophical views and personality traits that heavily influenced their careers and were contributing factors to their success.

An examination of Boole and Hamilton's lives sheds new light on old myths regarding such individuals. For instance, neither fits the stereotype of "boring" with which mathematicians are so oft labeled. Indeed, their exciting connections with some of the most prominent men of their day were enough to keep their lives interesting. Humorous accounts such as that of Hamilton hiding under a car to protect his books during a rainstorm (and unfortunately losing his hat in the process) helps one see Hamilton as more than just the mathematician who discovered quaternions. As their biographers have commented, he and Boole were truly remarkable men with interesting personalities who just happened to be mathematicians. This talk will attempt to convey a little of the spirit and enthusiasm of these men's lives and work that is so characteristic of both.

George P. H. Styan
McGill University

Frederick Augustus Porter Barnard (1809--1889), the tenth president of Columbia College (now Columbia University) and after whom Barnard College is named, observed that "The construction of magic squares has been practiced earlier than the period of authentic history and it has preoccupied the attention of the curious in every age, among them men of high scientific eminence."

We agree and in this talk, we focus on magic square matrices that are fully magic, i.e., the rows, columns, and the two main diagonals all sum to the same "magic sum." We concentrate on magic square matrices with 3 nonzero eigenvalues and with rank equal to 3, and on some associated postage stamps.

The magic square associated with the "luoshu matrix" was apparently first considered by the (mythical) Chinese engineer-emperor Yü the Great (fl. c. 21st century BC), while the magic square associated with the appears in Albrecht Dürer's well-known copper-plate engraving "Melencolia I," which dates from A.D. 1514. This engraving is depicted on a stamp issued by Aitutaki--Cook Islands and on a stamp from Mongolia.

We present some (apparently new) closed-form matrix formulas for the odd and even powers of a magic matrix A with rank equal to 3 and with 3 nonzero eigenvalues, such as the "luoshu matrix" L, the "Dürer matrix" D, and over six hundred essentially different 4x4 magic square matrices.

Ruediger Thiele
University of Leipzig

How did Euler change mathematics?

Abstract: On the one hand, Euler best represented the natural sciences in the middle of the 18th century; on the other hand he was known as analysis incarnate. Indeed, it was mathematics – especially the rising analysis – which served for Euler as the ground on which he started his investigations. From this viewpoint it is interesting to see the way in which he changed mathematics and our view of nature. This lecture briefly discusses some essential points of the transition.

Glen Van Brummelen
Quest University

Telling Time in 10^{th}-Century Baghdad: A New Instrument for Solar Timekeeping Comes to Light

The recent discovery by David King of an instrument for telling time by the altitude of the Sun is rewriting the story of medieval astronomical timekeeping. The device, by the early 10^{th}-century astronomer Nastulus, is a graphical solution to a common approximate formula for which many numerical tables exist, including a well-known set by al-Khwārizmī. However, Nastulus's instrument applies a shockingly sophisticated mathematical approach, producing times of day for Baghdad in error by no more than a few minutes. We shall place the instrument in context and work out some examples with replicas.

Bart Van Kerkhove
Vrije Universiteit Brussel

The historicity of mathematics: computer proof

Ever since Kuhn's classic study of 1962, questions about alleged revolutionary changes in the history of science have become commonplace. The philosophy of mathematics as we find it however, does still not allow for any kind of paradigmatic shift at all. Contemporary discussion about the possibility of mathematical revolutions invariably comes down to settling a sterile and purely semantical matter, whereby revolutions are defined into existence or else (mostly) defined away.

But couldn't this be more than just a matter of principle, and perhaps also (at least partly) be one of historical fact? To wit: could not more and proper meta-scientific inquiry help sort out more clearly similar philosophical questions, for the simple reason that they exhibit, after all, an element of historical contingency? We shall be arguing that the answer to this question is affirmative, while within philosophy of mathematics, traditionally time-less as it is, this domain remains virtually unexploited, and mostly even deemed unworthy of any serious consideration.

Moreover, we do not only consider exceptional, but by extension also other, normal instances of mathematical development, as historically conditioned (however partially), and for that reason essentially more than part of a mere cumulation of truth upon truth in the direction of the one ultimate and encompassive supertheory. Other than exclusively to this latter (arguably useful) fiction, that is, mathematical developments deserve it to be weighed against their proper, contemporary and contextual rationality.

A notable aspect of actual practice thus to be considered is that of mathematical explanation. It hinges on what convinces us mathematically: the mere running through a purely mechanical procedure, or something more, either by intuition or argument. A specific theme to be definitely addressed in this respect is that of the methodological role of the computer as either an inductive or a deductive complement in the process of mathematical proof construction. It should be clear that a lot is at stake. Generally speaking, computer (assisted) proofs have an acute impact on the issue of explanation in mathematics, which by extension raises questions as to the whole notion of progress within the discipline. Addressing this issue requires one to be explicit about the philosophical endorsement of the historicity of mathematics, which might indeed imply a serious challenge to common epistemological conception.

Jean-Philippe Villeneuve
Université de Montréal

From Cauchy's integral to Lebesgue's integral axiomatization: When a new interpretation becomes a reinterpretation

The purpose of this talk is to review some of the most important 19th century research on the mathematical notion of the integral to shine the spotlight on two important processes: the new interpretation and the reinterpretation of a mathematical notion. What differenciates these processes is that, in the first process, the same way of defining the initial notion is kept; in the second, it has completely changed. Note that these processes can be linked to the generalization and abstraction processes, but this is fodder for another talk.

To illustrate that, we will discuss Cauchy's integral (1823) on continuous functions defined as the limit of the Cauchy sums. We will then look at Riemann (posthume 1867) and Darboux (1875) who reinterpret the Cauchy integral by introducing a new way of defining the integral: the equality between the upper and lower integrals. We will also present Jordan's new interpretation (1892) of both integrals within the context of $\mathbf{R}^n$ and the new notion he had to develop: the measure.

Finally we will see how Lebesgue (1902) reinterprets the integral by axiomatizing it and, using step functions, introduces a new way of defining the integral on bounded functions. With his axiomatization, the Riemann integral is no longer an integral, but the Cauchy integral on continuous functions keeps this title.

Marina Vulis
CUNY

Arabic contributions to cryptography

This presentation will discuss the contributions to cryptography and cryptanalysis made by the Arabs. Arab scientists are said to invent cryptanalysis, the art of analyzing and breaking ciphers. Even the very word 'cipher' was introduced by the Arabs.

The discussion will include the description of the work of Abu Yusef Yaqoub ibn Ishaq Al-Kindi. Al-Kindi described cryptanalytic techniques and statistical cryptanalysis and classified the cipher types known at his time. He also foresaw computational linguistics by studying Arab phonetics and the relative frequency of letters, sounds, and their combinations.