CSHPM SCHPM

Canadian Society for History and Philosophy of Mathematics
Société canadienne d'histoire et de philosophie des mathématiques

Annual Meeting/Réunion Annuelle
University of Waterloo, June 4 – 6, 2005

Saturday June 4

Morning: MC 2035
10:00 Opening remarks by Pat Allaire, Secretary of CSHPM/SCHPM
10:15 Jim Tattersall - *Arthur Buchheim and an Interpolation Formula*
10:45 Amy Ackerberg-Hastings - *John Playfair in letters*
11:15 Francine Abeles - *Lewis Carroll's Visual and Formal Logics*
11:45 Amirouche Moktefi - *How did Lewis Carroll become a logician*?

12:15 – 1:45 CSHPM Executive Council Meeting MC 2035

Afternoon Parallel Session 1: MC 2035
4:00 Robert Bradley - *The Genoese Lottery and the Partition Function*
4:30 Ed Sandifer - *Euler's Calculus Texts*
5:00 Antonella Cupillari - *The sixty-fourth article of the Instituzioni Analitiche*

Afternoon Parallel Session 2: MC 2038
4:00 Thomas Drucker - *Serendipity in Mathematics*
4:30 David Bellhouse - *A War of Words in Pictures: the dispute between Montmort and De Moivre over the probability calculus*
5:00 Miriam Lipshutz-Yevick - *Paul Lévy and the Dichotomy between the Normal and other Stable Probability Limit Distributions*

6:00 – 7:00 Book Launch: *Mathematics and the Historian's Craft The Kenneth O. May Lectures*, M. Kinyon and G. van Brummelen, Glen (Eds.)

Sunday, June 5

Morning: MC 2035
10:15 David DeVidi - *Logical pluralism and the municipal by-laws of thought*
10:45 Robert Thomas - *Mathematics as a science*
11:15 Cameron Zwarich - *Philosophical Implications of Recent Work in Set Theory*
11:45 Jonathan Seldin - *Curry's Formalism as Structuralism*

Afternoon Parallel Session 1: MC 2035

4:00 Tom Archibald - *Mathematics and the First World War*
4:30 Roger Godard - *Convexity*
5:00 Ed Cohen - *The Iranian Calendars*

Afternoon Parallel Session 2: MC 2038

4:00 David Orenstein - *`Greenwich? Ca n'importe. Où est PARIS?' A local post-Conquest eclipse-based longitude calculation at Quebec*
4:30 Joel Silverberg - *The Mathematics of Navigation as Taught in Private Venture Schools, Academies, and Colleges in the New England Colonies, 1725-1850*
5:00 Craig Fraser - *Theoretical Cosmology and Observational Astronomy Circa 1930*

Monday, June 6

Morning: MC2035

10:15 Hardy Grant - *Greek Mathematics and Greek Science*
10:45 Alexei Volkov - *Geometrical diagrams and their didactical functions in traditional Chinese mathematics*
11:15 Alexander Jones - *Enigmas of the Keskinto Astronomical Inscription*

Special Session: History of Mathematics from Medieval Islam to Renaissance Europe

11:45 Glen Van Brummelen - *Al-Samaw'al and the Errors of the Astronomers: Where the Mistake Really Lies*

12:15 – 1:45 CSHPM/SCHPM Annual Meeting MC 2035

CSHPM/SCHPM Plenary Lecture DC 1350

2:45 – 3:35 Len Berggren - *Currents and counter-currents in the history of mathematics in medieval Islam*

Afternoon Special Session: MC 2035

4:00 Federica La Nave - *Bombelli and L'Algebra*
4:30 Odile Kouteynikoff - *Guillaume Gosselin, an algebraist in Renaissance France*
5:00 Jozsef Hadarits - *Diamonds, Rings, and Squares: Eastern Magic in Western Hands*
5:30 Christopher Baltus - *When is a Negative Really a Negative*?
6:00 Lawrence D'Antonio - *Number Theory from Fibonacci to 17th Century Safavid Persia: a question of transmission of knowledge*

Visit the CSHPM booth in the Exhibits area, Davis Centre, Great Hall on Saturday and Sunday from 9:30 to 4:00.

Abstracts – Alphabetical by Author
CSHPM Annual Meeting
June 4-6, 2005

FRANCINE ABELES, Kean University, NJ
Lewis Carroll's Visual and Formal Logics

Charles L. Dodgson (Lewis Carroll) first published his visual method in *The Game of Logic*, a book published in 1886, extending it ten years later in *Symbolic Logic, Part I*. Originally designed to teach the theory of inference in Aristotelian logic and to improve on the earlier diagrammatic methods of Leonhard Euler (1772) and John Venn (1880), Carroll's method has not been considered seriously as a visual logic system.

In the two parts of *Symbolic Logic*, the second part first published in 1977, Dodgson developed a formal logic in which he set down valid rules for making inferences.

In this paper, I will describe the methods he invented to mechanize reasoning in his formal logic, and demonstrate the superiority of his diagrammatic method over Euler's and Venn's methods.

AMY ACKERBERG-HASTINGS, Maryland U. College
John Playfair in letters

This talk is a progress report on an effort to locate, transcribe, and annotate all of the surviving correspondence written by John Playfair (1748—1819), Professor of Mathematics and then of Natural Philosophy at Edinburgh University. The letters reflect Playfair's wide-ranging interests in philosophy, geology, anthropology, chemistry, architecture, literature, and drama. They contain lists of his prominent friends: John Leslie, Dugald Stewart, William Robertson, John Rennie, Mary and Agnes Berry, Lord and Lady Minto, Archibald Constable, and so on. They share opinions on how the British government handled the American Revolution and Napoleonic Wars. And, woven throughout the correspondence, are references to Playfair's lifelong concern for mathematics and its history, including his analysis of observations made at Schehallien, papers prepared for the *Transactions of the Royal Society of Edinburgh*, and his supplement to the *Encyclopaedia Britannica*.

TOM ARCHIBALD, Simon Fraser University, Burnaby, BC
Mathematics and the First World War

Following the declaration of war in 1914, scientific communications were interrupted. This occurred formally, so that (for example) German journals were no longer sent to libraries or individuals in enemy states. There was also an informal side, with many individuals deciding to cease correspondence with colleagues on the other side. This talk will give some examples of consequences internationally both during and after the war, as well as discussing activities by a few people who chose to resist this path.

CHRISTOPHER BALTUS, SUNY Oswego
When is a Negative Really a Negative?

Well into the 17th century, works in algebra commonly gave the rules for arithmetic involving negatives but did not allow negative solutions to equations. This paradoxical situation signals a less than full acceptance of negative numbers. I believe that the arithmetic rules, especially for multiplication, were intended for polynomial multiplication involving subtracted terms. I will speak of long term developments in this line of thought, from Brahmagupta and Islamic writers up to John Wallis.

DAVID BELLHOUSE, Department of Statistical and Actuarial Sciences, University of Western Ontario, London, ON
A War of Words in Pictures: the dispute between Montmort and De Moivre over the probability calculus

In 1708 Pierre Rémond de Montmort published his book *Essay d'analyse sur les jeux de hazard*, an analysis of games of chance of the time using probability theory. Three years later Abraham de Moivre published his treatise *De Mensura Sortis* solving various problems in games of chance, again using probability theory. Montmort felt that De Moivre had plagiarized his work. In 1718 De Moivre published an expanded version of his original Latin treatise under the name *The Doctrine of Chances*. The dispute is described from surviving publications and letters. Both the *Essay d'analyse* and *The Doctrine of Chances* contain engravings that describe in pictures the nature and importance of their work. These pictures are analyzed in the context of the dispute.

LEN BERGGREN, Simon Fraser University
Currents and counter-currents in the history of mathematics in medieval Islam

Among recent changes in approach to history of mathematics in medieval Islam are: a widened definition of 'mathematics' to include its applications to such religious duties as the times and direction of prayer and taking mathematical instruments as serious witnesses to mathematical activity; an argument that in medieval Islam theory informed and supported practice to a much greater degree than in ancient Greece (*e.g.*, in arithmetic, architecture and astronomy); a role for Babylonian mathematics in the origins of Islamic algebra different from what has been supposed; a close study of the mathematical achievements and interactions of a number of individuals; understanding the history of an area such as magic squares which seems unrelated both to much of which came before it and (for some centuries) to any social context. We shall conclude with a consideration of the motivations of medieval and Renaissance Europe for their acquisition of medieval Islamic mathematics and how such motives affected what material was selected for acquisition.

ROBERT BRADLEY, Adelphi University, Garden City, NY
The Genoese Lottery and the Partition Function

In 1749, King Frederic the Great sought Euler's mathematical counsel concerning the establishment of a state lottery. The combinatorial issues involved in the analysis of this game of chance, known as the Genoese Lottery, piqued Euler's curiosity. As a consequence, he wrote four memoirs over the course of his career examining questions arising from this lottery. We will survey these works, paying particular attention to the use of the partition function in the second one.

ED COHEN, Ottawa, Ontario
The Iranian Calendars

We study the modern Iranian (Persian) calendar after first considering older Iranian calendars. In examining the modern Iranian (solar) calendar we also discuss arithmetical formulas necessary to convert it reciprocally into the Gregorian calendar.

There will moreover be a short investigation on the Islamic (or Muslim lunar) calendar vis à vis the modern Iranian calendar. A few centuries after the modern Iranian calendar was started, Omar Khayyám (1048-1131) studied the exactitude of the leap year and made important contributions. Finally, we examine the political situation of the 20th century, which changed the various calendars back and forth, eventually settling again on the main calendar.

ANTONELLA CUPILLARI, Penn State Erie, Station Road, Erie, PA
The sixty-fourth article of the Instituzioni Analitiche

In 1748, after ten years of hard work, Maria Gaetana Agnesi (1718—1799) published the first Calculus book designed for teaching and written in Italian: *Instituzioni Analitiche ad uso della Gioventu' Italiana* (Analytic Institutions for the use of the Italian Youth). In the introduction to her work, Agnesi wrote: . . . *when considering the Integral Calculus, the Reader will find a completely new method for Polynomials, which has not appeared anywhere else; it belongs to the famous and never sufficiently praised Count Jacopo Riccati, Nobleman very proficient in all sciences, and well known in the literary world. He wanted to do me the favor of letting me know about it [the method], favor that I did not deserve, and I want to give him, and the Public, the appropriate justice, as it should properly be done.* What was this new method, presented in the sixty-fourth article of the book? Was it really about polynomials? Is it as useful as Agnesi seemed to think?

LAWRENCE A. D'ANTONIO, Ramapo College, New Jersey
Number Theory from Fibonacci to 17th Century Safavid Persia: a question of transmission of knowledge

How much influence did Islamic mathematics have on Renaissance Europe and vice versa? This possible transmission of knowledge is an interesting and important topic for the historian. In this paper the question of transmission is examined with regard to selected problems in number theory, in particular the problem of congruent numbers. A congruent number k is an integer for which there exists a square such that the sum and difference of that square with k are themselves squares.

Congruent numbers can first be found in various works of classical Islamic mathematics, for example, in al-Karaji's early 11th century text, the *al-Fakhri*. Congruent numbers then resurface in the treatise *Liber Quadratorum* of Fibonacci. We then find congruent numbers in the influential 17th century work, *Khulasat al-Hisab* of Baha al-Din.

Was the work of Fibonacci known in the Islamic world? This is not easy to determine, since there is no direct reference to Fibonacci in Islamic sources. On the other hand, Edouard Lucas, in a major essay on Fibonacci, shows the existence of an intellectual thread, if not a clear historical thread, connecting Fibonacci and Baha al-Din.

To examine the problem of transmission, it is necessary to look at the cultural context for mathematics during the Safavid dynasty of 17th century Persia. The Safavid period represents, perhaps, the last flowering of classical Islamic science. Under the reign of the Safavid ruler Shah Abbas I, 1588-1629, a cultural renaissance occurred in the capital city of Isfahan. Especially important are Safavid accomplishments in the areas of mathematics, astronomy, scientific instrument making, carpet weaving, medicine, and architecture. Safavid mathematics is represented primarily through the work of Baha al-Din and Mohammad Baqir Yazdi (whose major work, the *Uyun al-Hisab*, also includes some interesting results in number theory).

It is well-known that many different Europeans spent time in the court of Shah Abbas. Adventurers, travelers, and missionaries were attracted by this center of learning. This paper examines possible sources of transmission. For example, it is known that the 17th century Italian traveler, Pietro Della Valle, did discuss current trends in astronomy with Persian scientists.

The discussion of the work of these Safavid scholars will hopefully contribute to a more complete picture of classical Islamic mathematics.

DAVID DeVIDI, Department of Philosophy, University of Waterloo, Waterloo, ON
Logical pluralism and the municipal by-laws of thought

Is there any sense in which it is both *interesting* and *true* that there is a plurality of logics? There is, of course, a multiplicity of systems traveling under the name 'logic': various modal, deontic, combinatorial, constructive, paraconsistent, relevant, higher order, free, and other 'logics', not to mention impoverished ancestors like Aristotelean syllogism that differ from the standard first-order predicate logic favored by mathematicians and philosophers. But for all that, there might be a plurality of logics in only a trivial or uninteresting sense.

In this paper the prospects for logical pluralism are investigated. In particular, currently popular defenses of pluralism, such as the one due to J. C. Beall and Greg Restall, are investigated and found to yield just such an uninteresting logical plurality. An alternative version of pluralism is sketched, beginning with the observation that a variety of traditional accounts of what distinguishes logical from non-logical principles, usually regarded as equivalent, actually draw the logic—non-logic line in different places.

THOMAS DRUCKER, University of Wisconsin-Whitewater, Whitewater, WI
Serendipity in Mathematics

Robert K. Merton's 'On the Shoulders of Giants' has had a good deal to offer the community of historians of mathematics as well as historians of science in general. His posthumous book on serendipity has come in for rather harsher treatment by both groups. This talk is designed to point out some of the theoretical features of the book which apply to mathematics and to illustrate how this fits into Merton's general view of the rationality of the scientific and mathematical enterprises.

CRAIG FRASER, Inst. Hist. Phil. Sci. Tech., Victoria College, University of Toronto
Theoretical Cosmology and Observational Astronomy Circa 1930

That the invention of geometric cosmological models based on general relativity occurred at the same time that Vesto Slipher and Milton Humanson were documenting systematic large nebular red-shifts seems to have been a coincidence. Edwin Hubble in 1936 explicitly associated the expanding-universe interpretation of his red-shift law with relativistic cosmology; for Hubble, universal expansion was a theoretical notion rooted in relativity theory. The paper explores various historical questions concerning the relationship between theory and observation in cosmology around 1930.

ROGER GODARD, Department of Mathematics Royal Military College of Canada
Convexity

By the middle of the 19th century, it was recognized that Euler's gamma function had some special properties. One of them will be convexity. A curve is convex if the following is true: take two points on the curve and join them by a straight line; then the portion of the curve between the points lies below the line. A convex function cannot look like a camel's back! It corresponds to a fundamental geometric concept of a function. In this work, we present some concepts developed by Jensen in 1906, in *Sur les fonctions convexes et les inégalités entre les valeurs moyennes*, and mainly Minkowski's work. We discuss some important applications of convexity in variational calculus, linear programming and non-linear programming.

HARDY GRANT, York University, Toronto
Greek Mathematics and Greek Science

The application of mathematics to the physical world was rather more problematic for the Greeks than it is for us. I shall here be concerned not with the achievements of the Greeks in this line, which are generally well known, but with the underlying conditions, the philosophical and cultural assumptions which tended to encourage or to inhibit the endeavour.

Relevant issues include the compartmentalization of knowledge, debate over the desirability and possibility of abstraction, and the widespread doubt that we can have true knowledge of the changeable.

JOZSEF HADARITS, Royal Ontario Museum
Diamonds, Rings, and Squares: Eastern Magic in Western Hands

In medieval Islamic mathematics there were two basic methods for constructing odd-order magic squares: the so-called "diamond" technique and another, more sophisticated one that can be understood in terms of a virtual torus. The West produced the first detailed description of the latter technique during the Renaissance period. Using historical evidence, including that of art, this paper makes an attempt to trace some of the possible routes of this intercultural scientific reception—in order to get closer to the understanding of the cosmological-spiritual background of these centuries-old mathematical problems.

ALEXANDER JONES, University of Toronto, Toronto
Enigmas of the Keskinto Astronomical Inscription

Just over a hundred years ago, a Greek stone inscription was discovered on the island of Rhodes, at a site near Lindos called Keskinto. The inscription, of which only the last fifteen lines are partially preserved, dates from about 100 B.C. and contains a set of periodicities for the motions of the planets. The relations between some of the numbers were explained by Paul Tannery shortly after the text of the inscription was published in 1895, but little progress has been made since in making sense of the astronomical and mathematical principles underlying the inscription. The present talk will discuss the problems and present some new tentative solutions.

ODILE KOUTEYNIKOFF, IREM, Université Paris VII Denis Diderot
Gulliaume Gosselin, an algebraist in Renaissance France

Guillaume Gosselin de Caen's treatise, known as *De Arte Magna* (Paris, 1577), is a short and quite simple work written by someone who is a typical algebraist in Renaissance France. Gosselin learned mathematics and heard about new methods in algebra from mathematicians who worked just before him; after making these new methods his own, he wanted them to be taught and wrote them down. He is especially good at solving problems with several unknown quantities and several linear equations.

It is important to notice that Gosselin's book is very dependant both on Italian Tartaglia's *Arithmetic* (Venise, 1556), which Gosselin translated into French by the same time he wrote *De Arte Magna*, and on Diophante's *Arithmetics*, which came to be known exactly two years before, thanks to Xylander's translation into Latin (Bâle, 1575). Both Gosselin and Tartaglia refer to Pacioli's work (*Summa*, Venise, 1494) and Pacioli himself says he learned much from Fibonacci, especially through *Liber Quadratorum* (Pise, 1225).

According to the fact that Al-Khwarizmi founded Algebra during the 9th century, it is not surprising that, when being translated into Arabic in the late 9th century by Lebanese Ibn Luqa whose native language was Greek, Diophante's *Arithmetics* seemed to be considered as a treatise about Algebra since algebraic vocabulary and way of thinking were most widely shared. Only few people understood that it was actually an arithmetic treatise: Al-Khazin (900—971) did, and therefore he is one of those who laid the foundations for the integer Diophantine analysis. We know that Jean de Palerme submitted Al-Khazin's problem about congruent numbers to Fibonacci, who then wrote *Liber Quadratorum*.

These are the main ways that lead from Diophante, as both a Greek and an Arabic source, to Renaissance Europe readers. We will show from his text how eager to learn and respectful of what he learnt Gosselin was, and how enthusiastic about new algebraic methods he was too. He wished he could retranslate and explain the complete Diophante's *Arithmetics*, but he didn't. We ignore what kind of work he would have done, either an exact arithmetic treatise as Bachet (Paris, 1621) and Fermat (1601-1665) did, or an up-to-date algebraic one according to what all algebraists did, such as Bombelli (*Algebra*, Bologne, 1572), Stevin (*Arithmetic*, Leyde, 1585), Viete (*l'Art Analytique*, Tours, 1591-1593) or Girard (*L'invention nouvelle en Algèbre*, Amsterdam, 1629).

FEDERICA LA NAVE, Harvard/Dibner Institue
Bombelli and L'Algebra

In *L'Algebra*, Bombelli was the first to recognize what we call "imaginary numbers" as numbers, and to give them operative definitions. *L'Algebra* was published in three books in 1572. However, Bombelli wrote the first version in five books in 1550. In the 1550 manuscript version Bombelli does not believe that the roots born in solving cubic equations in the irreducible case are numbers. In the published version he believes they are numbers and gives rules for operating with them. The aim of this paper is to try to understand what happened in these twenty-two years to Bombelli's state of belief—how his beliefs changed and what caused that change.

MIRIAM LIPSHUTZ-YEVICK, Rutgers the State University (Retired); 22 Pelham Street, Princeton, NJ
Paul Lévy and the Dichotomy between the Normal and other Stable

The ubiquity of describing the statistics of characteristics in large populations by a normal distribution is commonly accepted. Thus for instance Hernstein and Murray in their widely disseminated book *The Bell Curve* describe the Normal Distribution as:

> "A common way in which natural phenomena arrange themselves approximately."

In fact their model for the distribution of IQ's is mathematically way off the mark in satisfying the criteria for a normal distribution.

A more precise, yet still off the mark definition is in Jim Holt's article in the January 4, 2005, New Yorker Magazine:

> "As a matter of mathematics the Bell Curve is guaranteed to arise whenever some variable is determined by lots of little causes (like human height, health, diet) operating more or less independently."

The great French mathematician Paul Lévy in writing his classic 1924 *Calcul des Probabilités*—in spite of the fact that the eminent mathematicians Borel and Deltheil felt it unnecessary to make the notion of probability more mathematically precise rather than to rely on common sense reasoning—intended to systematically develop and use the method of characteristic functions in order to simplify proofs about limit laws.

A sufficient condition for the sums of a large number of "individually small", independent random variables to approach the normal distribution—*i.e.*, for the Central Limit Theorem to hold—was first given by Liapounoff in 1901 and a more general one
Lindeberg in 1922. Paul Lévy used this simpler method to derive Lindeberg's condition. In so doing he put his finger on the essential necessary condition and its meaning for the Central Limit Theorem to hold. This condition states that the components of the sum be not only "individually negligible" (small) with respect to their total sum, but that they be "uniformly negligible" (we might use the term "collectively negligible"), *i.e.*, the probability that even the largest component random variable be of the order of the magnitude of the sum, must be negligible.

Lévy showed that in case this condition is not satisfied there exist families of limiting distributions for sums of independent random variables, among which the so-called Stable Laws of Index α, where $0 < \alpha < 2$. Here the approach to the limiting distribution is determined by the contribution of the few largest components in the sum. Consequently the probability of values of the sum which deviate from the mean (or the median in case $\alpha < 1$) by a large amount is considerable. The “tail” of the probability distribution of the largest component in the case where the sum converges to a stable distribution of index α, as well as the ‘tail” of the limiting stable distribution decrease as the function $x^{-\alpha}$. The limiting distribution of the sum reflects that of its largest component terms. This dichotomy defines the “domains of attraction” of the normal vs. those of the other stable distributions.

The statistics of social phenomena in which stable distributions prevail such as wealth, power, batting averages, intellectual accomplishments, physical beauty, etc., are hardly ever discussed in the popular culture where they most emphatically deserve more attention.

AMIROUCHE MOKTEFI, IRIST (Strasbourg)/LHPS (Nancy), France
How did Lewis Carroll become a logician?

It’s well known that Lewis Carroll, the famous author of the Alice books, was a mathematician. His works include essentially manuals of Euclidean geometry, a treatise on determinants, popular textbooks on logic and collections of problems and puzzles. The majority of these works were signed with his real name: Charles L. Dodgson. The logical works are an exception. In effect, Carroll signed his two textbooks *The Game of Logic* (1886) and *Symbolic Logic* (1896) and his two articles in Mind: “A logical Paradox” (1894) and “What the Tortoise said to Achilles” (1895) with his “literary” pseudonym.

This state of affairs led to a number of prejudices and misunderstandings which influenced the reception of the work. People thought that Carroll’s work was intended for children, that he considered logic a game and that his logical work completed and concluded his literary work. Even when his works revealed accurate ideas and discoveries, commentators claimed Carroll was not fully “conscious” of the depth of his works.

In this paper, I will essentially try to contextualise Lewis Carroll’s logical work according to three themes: first, historically according to the development of the new logic in the nineteenth century; then geographically, by focusing on the academic British background; and finally personally by situating Carroll’s logical works in relation to the rest of his work. From this, we can correct certain received ideas which harm the understanding of the work. Also, we will be able to suggest new ways to explain the use which Carroll made of his pseudonym, the growing interest which he had in logic, and finally the status he gave it.

DAVID ORENSTEIN, Toronto
‘Greenwich? Ca n’importe. Où est PARIS?’ A local post-Conquest eclipse-based longitude calculation at Quebec

One of the teaching fathers at the Séminaire de Québec observed the October 27, 1780, solar eclipse. In an unsigned manuscript attributed to Thomas-Laurent Bédard, superior of the Séminaire, he followed the methods of Lalande’s ASTRONOMIE, using the copy from the seminary’s library, still on site in the archive. From this detailed observation he calculated the longitude of Quebec relative to Paris, not Greenwich, England, despite the British Conquest.

ED SANDIFER, Western Connecticut State University, Danbury, CT
Euler’s Calculus Texts

Euler’s four-volume 2500 page calculus text is often described as the origins or foundations of the modern calculus curriculum. This idea should not be accepted without some reflections. For example, few modern mathematics curricula include properties of elliptic integrals, as does Euler’s Integral Calculus. We describe the content and intent of Euler’s calculus course, and make some comparisons with the modern curriculum.

JONATHAN SELDIN, University of Lethbridge, Lethbridge
Curry's Formalism as Structuralism

H. B. Curry is known for a philosophy of mathematics which he called formalism". However, most people who know anything of his philosophy identify it with an early version which dates to 1939, relatively early in his career. In this early version, Curry proposed that mathematics be defined as the "science of formal systems", where he had in mind a definition of formal system somewhat different from the usual notion. Among the criticisms of this proposed definition are the suggestion that under this definition there could have been no mathematics before there were formal systems, a little more than a century ago.

In this paper, I will quote from Curry's later work to show that this criticism does not apply to his mature philosophy, and that his mature version of formalism is a form of structuralism.

JOEL SILVERBERG, Roger Williams University, Bristol, RI
The Mathematics of Navigation as Taught in Private Venture Schools, Academies, and Colleges in the New England Colonies, 1725—1850

Curricula and textbooks used in colleges, academies, and private schools in the newly formed United States between 1776 and 1826 show a gradual evolution from a vague exposure to whatever mathematical works, primarily of British origin, were in the possession of tutors, teachers, and faculty, to a more structured plan in which students progressed through arithmetic, algebra, geometry, and trigonometry, culminating in an application of these areas of mathematics to navigation and surveying, supported by texts specifically written for that purpose.
While these navigational topics can be found in documents dating from the earliest years of the 18th century, reflecting the study of navigation from private tutors and almanac makers, by the second decade of the 19th century the teaching of navigation had divided into at least two paths: one for professional mariners typified by the work of Nathaniel Bowditch, and a second as a capstone liberal arts experience (intended to teach scientific and mathematical principals) at colleges and universities, exemplified by texts written by Jeremiah Day, Professor of Mathematics and eventually President at Yale University.
This talk will present the details of these curricula and the ways in which navigation was used to illuminate the principles of Geometry and Trigonometry for the students of early America.

JIM TATTERSALL, Providence Coflege, Providence, RI
Arthur Buchheim and an Interpolation Formula

Arthur Buchheim was a short-lived mathematician of great promise. He attended the City of London School when Edwin A. Abbott (*Flatland: A Romance in Many Dimensions*) was headmaster. Buchheim received his undergraduate degree from Oxford. He left England for a time to study under Felix Klein in Leipzig. Upon his return, he accepted a position as mathematical master at the Manchester Grammar School. In the short span of seven years, and in deteriorating health, he published twenty-four papers on a wide variety of mathematical topics. Sylvester claimed that "had his life had been spared, I think we may safely say of him what Newton said of Cotes, that if he had lived, we should have known something." We focus our attention on Buchheim's work and accomplishments.

ROBERT THOMAS, University of Manitoba, Winnipeg, MB
Mathematics as a science

While mathematics can be regarded as both a science and as an art and benefits from the tension between those two motivations for practising it, philosophies of mathematics often do not take either of these often defended views with much seriousness. While I am unable to offer philosophical elaboration of the art view (though not wishing to disparage it), I present how it is that I see my science view of mathematics as connected to similar views of other sciences.

GLEN VAN BRUMMELEN, Bennington College
Al-Samaw'al and the Errors of the Astronomers: Where the Mistake Really Lies

Ibn Yahya al-Maghribi al-Samaw'al, a 12th-century converted Jew most known for his contributions to an arithmetical revolution in algebra, also wrote an intriguing but rarely studied book entitled *Exposure of the Errors of the Astronomers*. In it he takes shots at many of his predecessors, as far back as Ptolemy, for choices that they had made in their astronomical methods. Some of his criticisms seem odd, almost off the wall to a modern reader, but perhaps there are lessons here for an understanding of the medieval scientific mind. We shall explore some of his criticisms and attempt to put into historical context the rationality behind his criticisms.

ALEXEI VOLKOV, UQAM, Montréal, Québec
Geometrical diagrams and their didactical functions in traditional Chinese mathematics

The paper focuses on the use of diagrams in traditional Chinese mathematics, *i.e.*, the mathematical tradition that existed in China before the introduction of the corpus of mathematical literature by Jesuits and their local collaborators in the 17th century. The central questions to be addressed in the paper are:

(1) how the geometrical objects were perceived in traditional Chinese mathematics, and

(2) what were the modes of use of the diagrams in traditional mathematics education?

It appears that there existed several different stages at each of which the diagrams in mathematical books played different parts. My analysis shows that the earliest known Chinese mathematical diagrams of the first millennium AD were mainly focused on "depicting the structure" of mathematical objects. However, for various (and still unknown) reasons the diagrams were almost entirely lost by the end of the first millennium AD. The earliest extant printed diagrams (of the early 13th century) probably reflect at least one particular aspect of this tradition, namely, close interrelationship between geometrical and algorithmic (or algebraic) considerations. It appears that some mathematicians of the early first millennium AD attempted at perpetuating the original approaches to the diagrams, while other authors tried to avoid recourse to diagrams as to vehicle of argumentation. As result, by the 14th century the "conceptual diagrams" of the forefathers of Chinese mathematical tradition started giving way to a tradition of "naïve" representation of geometrical objects examples of which can be found in the later books on "popular mathematics".

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Philosophical Implications of Recent Work in Set Theory

Cohen's method of forcing allows one to show that many concrete mathematical statements are undecidable from the axioms of ZFC (Zermelo-Fraenkel set theory with the Axiom of Choice). A natural question arises: Can one repair the weaknesses in ZFC exposed by forcing, and if so, to what extent? The investigation of this question involves the study of higher axioms of infinity (the so-called "large cardinal axioms"), their canonical models, and the determinacy of infinite games. Recently, Woodin has developed Ω-logic, a strong extension of first-order logic that is the natural logic given by the method of forcing. He has also developed a transfinite proof system for Ω -logic, and isolated the Ω Conjecture, which is essentially completeness for this logic and its corresponding proof system. A proof of the Ω Conjecture would quantify the limits of forcing, provide a possible solution to the Continuum Hypothesis, show that those large cardinal axioms which admit an inner model theory of the kind that we know today are "cofinal" amongst all large cardinal axioms, and challenge the popular conception that there is no need for additional axioms of set theory.