Annual Meeting/Réunion Annuelle

The Canadian Society for History and Philosophy of Mathematics/ Société canadienne d'histoire et de philosophie des mathématiques

University of Toronto, May 24-26, 2002

Thursday, May 23, 2002

Special meeting in honor of the retirement of Israel Kleiner
York University, 1:30 to 5:30
Speakers: Craig Fraser and Pat Rogers
Reception following

Friday, May 24, 2002

Special Session in Memory of Ken May Emmanuel College 001

Chair: Craig Fraser

9:00	I. Grattan-Guinness, *Lecture in Bilbao History or genealogy? Historians and mathematicians on the history of mathematics*
10:00	Albert Lewis, *Kenneth O. May and Information Retrieval* (read by I. G-G.)
11:00	break
11:15	Francine Abeles, *Kenneth O. May on Arrow and Group Choice*
11:45	Thomas Drucker, *Ken May in the Classroom*
12:15	Greg Moore, *Ken, Communism, and Canada: Reminiscences about Seven Years as a Graduate Student of K. O. May*
12:45-2:45	Lunch break and Executive Committee meeting

General Session I Emmanuel College 001

Chair: Robert Bradley, John Glaus, and Ed Sandifer

2:45 Ed Sandifer, *Reciprocal Trajectories and Euler's Marvelous Geometry Machine*

3:15 John Glaus, *Jean le Rond d'Alembert "L'Enfant terrible du Siecle des Lumieres"*

3:45 Robert Bradley, *Priority Disputes and the Euler-d'Alembert Correspondence*

4:15 break

4:30 Antonella Cupillari, *Rules of Differentiation: Learning from Leibnitz and Agnesi* (read by P. A.)

5:00 Lawrence D'Antonio, *Henry Smith's 1875 paper "On the Integration of Discontinuous Functions"*

Party at Craig's Place!

7:00 94 Arundel Avenue, Chester subway stop on the Bloor-Danforth line.

<u>Saturday, May 25, 2002</u>

General Session II Victoria College 323

Chair: Hardy Grant

8:45 Duncan Melville, *Some Old Babylonian Geometric Figures*

9:15 Edward Cohen, *The Mayan Calendars*

9:45 Hugh McCague, *the Geometry of the Roman* Groma *Land Surveying Instrument*

10:15 break

10:30 Amy Ackerberg-Hastings, *An Illustrated History of the Protractor*

11:00 Barnabas Hughes, *The Geometric Algebra of Leonardo Fibonacci, Pisano*

11:30 Jim Tattersall, *George Airy: The Conservative Lucasian*

12:00-1:00 Lunch, Ned's Café (across St. Charles St.)

1:00-2:00 Annual General Meeting, V.C. 323

General Session III (Philosophy) Victoria College 323

Chair: Pat Allaire

2:00 Rebecca Adams, *A Comparison of Metrization Theorems*

2:30 Bart Van Kerkhove, *Guises of Naturalism in the Foundations of Mathematics Debate*

3:00 David Laverty and Gregory Lavers, *Frege, Carnap, and Conceptual Analysis*

3:30 Jonathan Seldin, *Curry's Anticipation of the Types Used in Programming Languages*

4:00 break

4:15 Madeline Muntersbjorn, *Who Does Mathematics?*

4:45 Catherine Womack, *Visual Reasoning in Mathematical Proofs: From Bolzano to Cantor and Beyond*

5:15 Yvon Gauthier, *Kronecker's Programme in the Foundations of Mathemaitcs*

<u>Sunday, May 26, 2002</u>

General Session IV Victoria College 211

Chair: Amy Shell-Gellasch

8:15 Massimo Mazzotti, *The Mathematics of Modernization: The Training of Engineers in 19th-Century Italy*

8:45 Christopher Baltus, *The Last Article of Gauss's First Proof of the Fundamental Theorem*

9:15 Israel Kleiner, *Fermat: The Founder of Modern Number Theory*

9:45 break

10:00 Patricia Allaire, *Have You Seen This Man? Where was Robert Murphy Between 1833 and 1836?*

10:30 Barry Davies, *Reimann's Tensor as Oral Tradition*

11:00 Tom Archibald, *Charles Hermite as a Mentor*

11:30 Hardy Grant, *The Resolution of Gauss's Class-Number Conjecture*

12:00-1:15 Lunch

Special Session on Numerical Mathematics Victoria College 211

Chair: Rebecca Adams

1:15 Roger Godard, *Numerical PDE: an Historical Sketch*

1:45 Erwin Kreyszig, *From Classical to Modern Numerics*

2:15 R. Bruce Simpson, *The Remarkable History of FORTRAN*

Joint Session with the Canadian Society for the History and Philosophy of Science

Proof, Prediction and Mathematics in Ancient and Islamic Science /
Preuve, prédiction et mathématiques dans les sciences antiques et islamiques

Funded by: The Humanities and Social Sciences Federation of Canada /
La Fe'de'ration canadienne des sciences humaines et socials

Victoria College 323

Chair: Craig Fraser

3:15 Alan C. Bowen, *The Exact Sciences in Fourth-Century Greece and their Interpretation in Aristotle's* Posterior Analytics *1.13*

3 :40 Daryn Lehoux, *Predictions in Ancient Astronomy and Astrology*

4 :05 Alexander Jones, *Frames of Reference in Ancient Astronomy*

4 :30 Glen Van Brummelen, *Analysis, Synthesis, and Computation: From Pure Geometry to Applied Mathematics in Medieval Islam*

The Canadian Society for History and Philosophy of Mathematics — May 24-26, 2002
Société canadienne d'histoire et de philosophie des mathématiques — 24-26 mai 2002

ABSTRACTS

Special Session in Memory of Ken May

Kenneth O. May on Arrow and Group Choice
Francine Abeles, Kean University

In 1952 Kenneth O. May published the article, "A Set of Independent Necessary and Sufficient Conditions For Simple Majority Decision," where he discussed group choice by simple majority vote in the context of Kenneth J. Arrow's watershed publication, *Social Choice and Individual Values* (1951). May left unresolved the restrictions necessary on individual preferences in order that group preferences be transitive. I will report recent work by Donald G. Saari (2000) that resolves these issues.

Ken May in the Classroom
Thomas Drucker, University of Wisconsin at Whitewater

Ken May spent much of his later life at the centre of developments in the history of mathematics community, both institutional and written. In the midst of this work, he also continued to teach a survey course on the history of mathematics which introduced the subject to beginners as well as to those who had come to the Institute for the History and Philosophy of Science and Technology to work with him. His flexible design of the course has been a model for many of those who had the pleasure of taking it. This talk will look at how the history of mathematics took shape in the classroom and how the discussions continued outside the walls of the classroom as well.

Lecture in Bilbao History or genealogy? Historians and mathematicians on the history of mathematics
I. Grattan-Guinness, Middlesex University at Enfield

Mathematics shows much more durability in its attention to concepts and theories than do other sciences. for example Galen may not be of much use to modern medicine, but one can still read and use Euclid.

One might expect that this situation would make mathematicians sympathetic to history, but quite the opposite is the case; As Philip Davis puts it, they despise history unless a priority dispute is at hand. Their normal attention to history is with genealogy; that is, how did we get here? Old results are modernized in order to show their current place; but the historical context is ignored and thereby quite distorted. By contrast, the historian is concerned with what happened in the past, whatever be the modern place. The difference between these two approaches will be discussed, with examples exhibited: these will include Euclid, set theory, limits, and applied mathematics in general. The implications for mathematics education will also be aired.

Kenneth O. May and Information Retrieval
Albert Lewis, Peirce Edition Project, Indiana University

In the 1960s and into the 1970s Kenneth O. May led a team of University of Toronto staff and students that aimed to produce the most comprehensive general reference tools to date for the history of mathematics. The major published result was the 1973 {\it Bibliography and Research Manual of the History of Mathematics}, but also in the works when he died in 1977 was a dictionary and thesaurus of mathematical terms linked to their use in the literature. May made the determination that using computers for his projects was not feasible at that time. When the American Mathematical Society investigated the possibility in the 1990s of converting his work into electronic form this proved not possible for a number of reasons. May's conceptual design was not at fault, indeed his goals and methodologies still stand as a model for information retrieval in this field.

Special Session on Numerical Mathematics

Numerical PDE: an Historical Sketch

Roger Godard, Royal Military College of Canada

In the numerical partial differential equations, a problem of Analysis is replaced by a problem of Algebra. 80% of the computer time consists in solving systems of linear equations. We emphasize the tremendous impact of least squares on the numerical linear algebra and the domination of the German school of Mathematics with Gauss (1810, 1817), Jacobi (1841), Von Seidel (1874), Liebmann (1918). The theoretical work of Nekrasov (1885) and Von Mises (1929) represent the first steps towards the processes of convergence of iterative systems of linear equations. Later one came computer arithmetic and the discovery of round-off errors in matrix processes (Turing, 1947).

The fundamental principles of finite differencing for partial differential equations are found in Hugo Buchholz' book (1908) who followed Ludwig Bolztmann's ideas, and L.F. Richardson (1910, 1928), and also Southwell (1935, 1940). The variational methods were presented by Rayleigh (1894, 1896) and independently by Ritz (1909) followed by Courant (1925). The fundamental paper on the numerical treatment of hyperbolic equations was published by Courant, Friedrich and Lewy in 1928. Therefore most of the classical numerical methods were discovered before the spread of the digital computer.

From Classical to Modern Numerics

Erwin Kreyszig, Carleton University

Most books and articles on numerics that appeared after the advent of the computer on the market, say, in the 1950s, show a general character distinctly different from that of the relevant publications in precomputer times. Almost all of the sudden, general ideas, such as stability and robustness of methods, became of central importance. Approximation and algorithm emerged as essential keywords. This also had a growing impact on technology-oriented changes of mathematical university education.

In this paper we shall explore historical roots of this evolution. We shall see that some of its ideas had forerunners in classical works by Newton, Legendre, Gauss, Jacobi, and others, that certain concepts gained greater prominence, and that methods impractical by hand or on a mechanical calculator were implemented in software. This will be illustrated by examples from interpolation, round-off and stability, and least squares.

The Remarkable History of FORTRAN

R. Bruce Simpson, University of Waterloo

What claims could this old dinosaur of computer programming have to being remarkable? I propose that high among them would be the basic historical 'facts' that it was:

- one of the first higher level languages to achieve commercial success (commencing about 1957)
- the first for which a language standard was formulated (1966)
- able to accommodate the software development trends of the 1970's
- the first language to revise its standards (1977)

I suggest that these facts have played key roles in the remarkable longevity of FORTRAN, maintaining its vitality for roughly 40 years. Some of the more qualitative aspects of these claims might be disputed. But if they are accepted, even in substance, they raise more questions than they answer.

- How does a programming language gain acceptance, commercial success?
- How do language standards influence the life of a programming language?
- Why is FORTRAN in decline?

and, even more fundamentally,

- Is it meaningful to generalize about programming languages?

We will present answers and speculations about these questions specific to FORTRAN that, in my view, make its history remarkable.

Joint Session with CSHPS on Ancient Science

The Exact Sciences in Fourth-Century Greece and their Interpretation in Aristotle's **Posterior Analytics** *1.13*

Alan C. Bowen, IRCPS

Usually one should segregate questions about the meaning of what ancient philosophers say regarding the exact sciences from historical questions about the truth of their claims. But this rule fails when the philosophical remarks are insufficient to allow a sure grasp of what was meant. Here, we must turn to what is known of the contemporary sciences to delimit possible interpretations. This is the case with Aristotle's comments about mathematics and the sciences that use mathematics in *Posterior Analytics* 1.13. To understand him, we must determine what the exact sciences were like in his time, and then proceed tentatively on the charitable assumption that what he says is true. Accordingly, I will first argue that there are but three surviving sources of the exact sciences in Aristotle's time: Aristoxenus' *Elements of Harmonia*, his *Elements of Rhythm*, and Hipparchus' quotations of Eudoxus' *Phaenomena*. This will entail demonstrating that the common practice of including Euclid's works among the scientific documents of the fourth century is based on a faulty appreciation of the evidence for his dates and of the signs in his treatises of later developments. Next, in light of these sources, I will propose that, when Aristotle says, for example, that optics is subordinate to geometry because it concerns the fact that something is the case and geometry gives the reason why, he means not that optics is applied geometry, but that optics often (but not always) draws on geometry to make its inferences, whereas geometry never draws on optics.

Frames of Reference in Ancient Astronomy

Alexander Jones, University of Toronto

Modern astronomy follows Ptolemy (mid second century A.D.) in taking the Spring Equinoctial Point (one of the intersections of the ecliptic--the plane of the sun's orbit--with the plane of the earth's equator) as the zero point for measuring planetary positions along the ecliptic. In Ptolemy's astronomy this frame of reference is clearly distinguished from the more "natural" frame of reference of the stars, which

Ptolemy considers unsuitable on theoretical grounds. We know that in Babylonian astronomy the two frames of reference (respectively called "tropical" and "sidereal") are not treated as distinct. An examination of computed positions of the moon and planets in Babylonian and Greek astronomical texts under certain conditions makes it possible to find out which was actually being used. One might have expected a fairly neat transition from purely sidereal, observation-based positions in Babylonian texts to purely tropical, computation-based positions after Ptolemy; the reality turns out to have been considerably more complicated.

Predictions in Ancient Astronomy and Astrology
Daryn Lehoux, University of King's College

In looking at the mechanisms of astrological prediction, we see that by relying on authoritative texts and instruments, the ancient astrologer was able to forecast the fates of individuals, as well as events such as meteorological phenomena and crop yields. This tradition finds its origins in several different omen traditions, common throughout the ancient Mediterranean and Near East, where different kinds of fortuitous events (including astronomical events such as eclipses) frequently had ominous significance. By the fifth century B.C., however, astronomy distinguished itself from the other omen traditions by developing mathematical methods for predicting the events (e.g., eclipses) from which its omens were derived. But the very adoption of these predictive methods served to canonize the timing and character of the astronomical events. That is: instead of being, strictly speaking, predictive, the texts and tools of early mathematical astronomy were normative. This meant that in making the astronomical part of their predictions, the astronomer, in spite of his rhetoric to the contrary, is primarily working from texts or instruments, rather than from observations in the natural world.

Analysis, Synthesis, and Computation: From Pure Geometry to Applied Mathematics in Medieval Islam
Glen Van Brummelen, Bennington College

The paired techniques of analysis and synthesis, born in ancient Greece, provided both a direct means to find a path to the solution of a geometrical problem and a way to convert that path into a rigorous proof. In medieval Islam analysis was used in this way, but it was also put to use in the solution of applied problems. We shall concentrate on texts by 10th century geometer Abu Sahl al-Kuhi and 12th-century mathematician Al-Samaw'al, to witness the transformation of geometric analysis to solve the problems of the distance to the shooting stars and the dip angle to the horizon for an elevated observer.

General Session

An Illustrated History of the Protractor
Amy Ackerberg-Hastings, Independent Scholar

The innocuous protractor has been a vital tool in engineering drawing and applied mathematics since the seventeenth century. In the spirit of Henry Petroski's *The Pencil: A History of Design and Circumstance* (1989) and Asa Briggs's *Victorian Things* (1989), I will discuss various historical forms and uses of this mathematical instrument. The talk will be illustrated with examples from the Smithsonian National Museum of American History's collections.

A Comparison of Metrization Theorems

Rebecca Adams, Orange Coast College

With an overview of 1917-1937, the work of Aleksandrov and Uryson (1923), "A Necessary and Sufficient Condition for an L-class to be a D-class," will be compared to Aline Frink's (1937), "Distance Functions and the Metrization Problem." I had the pleasure of visiting with Aline Frink (1991) and continued a correspondence with her until her death, March 14, 2000. The talk will include comments about her life as a personal tribute.

Have you seen this man? Where was Robert Murphy between 1833 and 1836?
Patricia Allaire, Queensborough Community College
It was in the writings of Robert Murphy (1806-1843) that William Thomson (Lord Kelvin) first learned of George Green's potential theory. During his brief career, Murphy published scholarly papers on algebra, difference equations, applied mathematics and the foundations of calculus, as well as textbooks on algebra and electricity. However, his "dissipated habits" resulted in the sequestration of his Cambridge fellowship in 1832. DeMorgan (and others) claim that Murphy spent the next several years in Ireland. There is evidence, however, that he remained at Cambridge and, therefore, had direct contact with Green. In this talk, in addition to considering Murphy's mathematical achievements, an attempt will be made to ascertain his whereabouts during the period in question.

Charles Hermite as a Mentor
Tom Archibald, Acadia University

In this paper I will discuss Hermite's role in influencing the direction of mathematical research in France in the late nineteenth century. From his appointment to the Académie des Sciences in 1856, Hermite was in a position to effectively promote research that he thought important. His influence grew with his mathematical reputation, notably following his elliptic-function solution of the quintic, but was doubtless greatest once he was appointed as a professor in the Faculté des Sciences in Paris. We shall discuss his direct and indirect influence, concentrating on the work of several doctoral students who were later to be influential.

The Last Article of Gauss's First Proof of the Fundamental Theorem of Algebra
Christopher Baltus, SUNY Oswego

In the final article of his 1799 dissertation, Gauss offered a second argument that, for any polynomial P(z), at least one curve Re(P(z)) = 0 meets a curve Im((P(z)) = 0. Background for this argument will be discussed, together with a suggestion that Gauss's thinking involved the Cauchy-Riemann Equations.

Priority Disputes and the Euler-d'Alembert Correspondence
Robert Bradley, Adelphi University

Leonhard Euler (1707-1783) and Jean le Rond d'Alembert (1717-1783) began a lively and fruitful mathematical correspondence in 1746. In 1751, however, d'Alembert angrily accused Euler of taking credit for some of his own discoveries, and broke off the correspondence for almost 12 years. In this talk, we examine the sources of discord between Euler and d'Alembert: disagreements over matters of mathematical truth (for example, in the logarithm of negative numbers), mathematical interpretation (the wave equation), and matters of priority (precession of equinoxes).

The Mayan Calendars
Edward L. Cohen

In the sixteenth century, the Spaniards conquered and persecuted the Mesoamerican (i.e., Central American) and South American civilizations. Among these cultures were the Maya and Aztec of Mesoamerica and the Inca culture of western South America. The Aztecs had their capital in Tenochtitlan (now Mexico City) and the Incas had their capital at Cuzco, Peru. Both the Aztec and Inca were recent occupants of their land—from about two centuries before; whereas the Maya had a long history divided into the following periods: *The Preclassic*: 200BCE – 250CE; *The Classic*: 250CE – 909CE; *The Postclassic*: 909CE – 1697CE. The Maya occupied what we now call Guatemala, Belize and portions of what we now call Mexico, El Salvador and Honduras. We now concentrate on the Maya calendars, which developed over a longer period; hence, deeper insights, particularly into astronomy, were expected.

Most articles on the Mayas have some mention of the Mayan calendars because of the calendars' importance to their everyday life. We shall study the work on their calendars by several scholars who deciphered it, especially in the last half of the twentieth century. It is hoped that an understanding of the Mayan calendars may be more appreciated. Extra references may be found in the bibliography at the end of the completed paper.

Rules of differentiation: Learning from Leibniz and Agnesi
Antonella Cupillari, Penn State Erie

We usually talk about "standing on the shoulders of giants" to give recognition to the great mathematicians who taught us so much. What can we learn from the giants when they either make a mistake or overlook essential details? Luckily, there is still plenty to consider in this cases. Let's consider briefly the first attempt by Leibniz to present the product and quotient rules and the detailed presentation of the rules of differentiation offered by Agnesi in the first calculus book designed as a teaching tool, the *Instituzioni Analitiche ad uso della Gioventu' Italiana.*

Henry Smith's 1875 paper "On the Integration of Discontinuous Functions"
Lawrence D'Antonio, Ramapo College

H.J.S. Smith, the Savilian Professor of Geometry at Oxford University, in 1875 wrote a ground-breaking, if at the time little- read, paper on the Riemann integral. Smith was more noted for his work in number theory, but shows in this paper a remarkable understanding of the fundamental issues of analysis.

The history of nineteenth century analysis is marked by a series of struggles regarding the proper definition and relationships of the concepts of continuity, differentiability, and integrability. Foundational questions of the following sort are raised. Are all continuous functions differentiable? Are all derivatives integrable? How badly discontinuous can a function be and still be integrable?

It is this latter question that is at the focus of Hankel's 1870 paper on discontinuous functions. Hankel sets out a program to classify such functions. Hankel incorrectly believed that being at most what he calls "pointwise discontinuous" defines necessary and sufficient conditions for a function being Riemann integrable. Smith's paper is clearly a response to Hankel, illuminating several problems with the Riemann integral.

This present paper examines the sources of Smith's investigations (primarily that of Riemann and Hankel), the examples that Smith provides to show the flaws in Hankel's program (one such example uses a Cantor set, eight years before Cantor), and the relationship of Smith's paper to later work in integration theory (such as that of Volterra, Dini, and Thomae).

Henry Smith is an interesting figure in nineteenth century mathematics, certainly worthy of wider study. His work in analysis, not particularly well known, is hopefully better illuminated by this paper.

Riemann's Tensor as Oral Tradition [*]
Barry Davies

When I was an undergraduate student of physics, the chair of the department believed that (at least once in their lives) all physics students should be able to derive Kepler's laws of planetary motion from Newton's laws. I propose here a candidate for a similar exercise in mathematics: At least once in their lives, perhaps, at least some mathematics students should be able to recall from memory Riemann's derivation of what came to be known as the Riemann-Christoffel tensor of the first kind. The derivation was a milestone in the history of mathematics, and it enables us to understand the possibility that space need not be Euclidean. In this paper the derivation is organized in a way that attempts to minimize its difficulty - so that it can be done (eventually) with the mind alone. This activity seems to be to the practice of mathematics what thinking ahead is to the playing of chess. Hence there seems to follow both the possibility and the desirability of the exercise as oral tradition.

Kronecker's programme in the foundations of mathematics
Yvon Gauthier, University of Montreal

Kronecker's work is generally ignored among historians and philosophers of mathematics and few mathematicians go beyond the common view about God-created integers and Kronecker's delta in their knowledge of Kroneckerian mathematics. Some mathematicians, may they be historians or not, among them Hermann Weyl, André Weil and Harold M. Edwards, have stressed the central importance of Kronecker's work not only in number theory and algebra, but also in algebraic geometry. I shall concentrate on Kronecker's major paper of 1882 « *Grundzüge einer arithmetischen Theorie der algebraischen Grössen* » (4) and emphasize the concept of a general arithmetic, that is the arithmetic of forms or (homogeneous) polynomials, in Kronecker's foundational stance.

I see Hilbert's programme as a direct heir to Kronecker's programme in the foundations of mathematics: the arithmetization of algebra, as Kronecker says, cannot dispense with the association of forms or indeterminates which Hilbert will call ideal elements « *Ideale Elemente* », except that Hilbert will grant some kind of (non-contradictory) existence to his ideal elements and then dispense with them in his finitist metamathematics, while Kronecker keeps his finitist outlook within general arithmetic. The internal logic of the general arithmetic of polynomials provides for a direct proof of the consistency of arithmetic. I have tried to put those ideas in the idiom of modern logic in a series of papers (1,2,3).

References

1. Gauthier, Y. " Hilbert and the internal logic of mathematics", *Synthese*, 101 (1994), no. 1, 1-14.
2. Gauthier, Y. " The internal consistency of arithmetic with infinite descent", *Modern Logic,* 8 (2000), nos 1/2, 47-86.
3. Gauthier, Y. *Internal Logic. Foundations of Mathematics from Kronecker to Hilbert,* Kluwer, Synthese Library, forthcoming in 2002.
4. Kronecker, L. "*Grundzüge einer arithmetischen Theorie der algebraischen Grössen",* in *Werke*, ed. by K. Hensel, 5 vols. Chelsea, New York, 1968, vol. III, 245-387.

Jean le Rond d'Alembert " L'Enfant terrible du Siecle des Lumieres"
John Glaus

This provocative title is not entirely misleading. Jean le Rond d'Alembert was an intelligent and distinguished scientist. He was responsible with Denis Diderot and others for the writing, editing and publication of l'Encyclopedie", the Enlightenment's holy book. This preliminary paper attempts to ferret out the personality who charmed Frederick II, conducted a twenty-seven year correspondance with Leonhard Euler and held court with Mme du Chatelet at her salon. References are from the newly translated "Letters of Leonhard Euler and Jean le Rond d'Alembert" Series Quarta A, Commercium Epistolicum Volume 5 by Robert E. Bradley and John Glaus.

The Resolution of Gauss's Class-Number Conjecture
Hardy Grant, York University

Quadratic forms of discriminant d fall naturally into equivalence classes, whose cardinality h(d) is called the "class number". Gauss conjectured that for negative d h(d) tends to infinity with -d, or (equivalently) that only finitely many negative discriminants correspond to a given class number. I shall try to sketch the eventual resolution of this conjecture, a tale that played out only in the 20th century and that presents several curious features.

The Geometric Algebra of Leonardo Fibonacci, Pisan.
Barnabas Hughes, California State University Northridge

Fibonacci's monumental *Liber Abbaci*, for its indepth development of some ninety problems and their solutions, added considerable impact to the growing appreciation of Arabic algebra. Regardless of being a linear successor of al-Khwarizmi, Abu Kamil, and al-Karkhi, he was fully committed to the thinking and use of geometry for algebra. This presentation will suggest that Leonardo deliberately brought geometry into his expansion of the algebra he received from his predecessors, to the extent that some problems seemed to require geometry to formulate the equations that would lead to algebraic solutions, not to overlook the need for geometry to substantiate the algebraic procedures.

Guises of Naturalism in the Foundations of Mathematics Debate
Bart Van Kerkhove, Vrije Universiteit Brussel

In the aftermath of the early 20th century foundational crisis in mathematics, a school of approaches can be identified emphasizing the significance of actual mathematical practice. This peculiar translation of the general and originally Kuhnian epistemological principle, viz. of taking an initial descriptive stance, to mathematics, was forwarded as an alternative to the "failing" mainstream (perfectibilist) answers. It indeed goes supported by empirical research focusing on human constraints, such as social structures, or feasibility and fallibility when doing real proofs. However, the application has come in different guises. In my talk, I shall present a framework capturing this diversity of naturalist mathematical accounts, viz. along an internalist-externalist or foundationalist-nonfoundationalist axis, as well as some ideas, mainly from cognitive science, that might lead to an integration of their advantages.

Fermat: The founder of modern number theory
Israel Kleiner, York University

I will discuss Fermat's contributions to number theory, noting his intellectual debts and his legacy.

Frege, Carnap, and Conceptual Analysis
David Laverty and Gregory Lavers, University of Western Ontario

In this paper we begin with a discussion of the motivating factors behind Frege's logicist program. We argue that Frege sought to refute Kant by providing an account of our knowledge of arithmetic which conforms to the pre-analytic intuitions we have on this subject. We claim that this due to the fact that for Frege there is a notion of "correctness" in conceptual analysis –that the end result of a conceptual analysis must cohere with our pre-analytic intuitions. We then contrast this with Carnap's logicism and his use of conceptual analysis. We claim that Carnap's Principle of Tolerance and his resulting views concerning the conventional status of our arithmetical knowledge illustrate that this notion of "correctness" in conceptual analysis is missing in Carnap's employment of the methodology. We next offer support for a methodology in conceptual analysis in which one does attempt to address the justification for pre-analytic intuitions. We argue that a reconstruction (given proper restrictions) can be used to answer philosophical problems. These restrictions involve conforming to pre-analytic intuitions. But as these intuitions are not entirely clear, they do not completely characterize what should count as true in a reconstruction. What does not clearly follow from or go against our pre-analytic intuitions is something that reconstruction must settle by convention. Thus, given the limited extent of such pre-analytic intuitions in arithmetic, we are driven to a view concerning the epistemological status of our arithmetical knowledge which, while not conventional to such an extreme as Carnap's, nonetheless has certain conventional aspects.

The Mathematics of Modernization: The Training of Engineers in 19th-Century Italy
Massimo Mazzotti, Kenneth O. May postdoctoral fellow, Institute for the History and Philosophy of Science and Technology

This paper explores the interaction between mathematical training, engineering practice and administration of the state. The professionalization of civil engineering is commonly understood as a consequence of a generic improvement in the mathematical training of engineers. Looking at the case of 19th-century Italy, this paper argues that engineers were not simply taught "more" mathematics. They were trained in specific forms of mathematical reasoning that -it was believed- would have supported best the "modernization" of the state. The emergence of the professional engineer, the legitimation of his "new mathematics", and the implementation of the liberal reform of the state should therefore be understood as aspects of a single historical process of social and cultural change.

The Geometry of the Roman Groma *Land Surveying Instrument*
Hugh McCague, York University

One of the main Roman land surveying instruments was the *groma.* It was used to simply sight straight lines and right angles, but nevertheless offers a wide range of geometric issues of historical note. Based on historic textual and archaeological evidence, essentially two different reconstructions of this instrument have been made. I will consider the mathematical issues involved with the two reconstructions, and the geometric elements of the design and use of the *groma* that were crucial to the accurate achievement of standard rectilinear land divisions or centuriation. Additionally, the collection of Roman land surveying manuals, the *Corpus Agrimensorum,* includes some geometric techniques and methods, without proofs, that show the use of the *groma* to sight straight lines not part of right angles. It also includes associated applications of basic laws of triangles, parallelograms, and even 3- dimensional geometry. Further, the geometry applied in the design and use of the *groma* was deemed to have a heavenly origin and was patterned after the divine archetype of the Roman *templum* of the sky. Though the use of the *groma* did not apparently continue during the medieval period, the *Corpus Agrimensorum* was formative in the later developments of practical geometry during the Middle Ages, particularly through such eminent scholars as Gerbert and Hugh of St. Victor.

Some Old Babylonian Geometric Figures
Duncan J. Melville, St. Lawrence University

One of the more complicated figures studied in Old Babylonian mathematics is the *apsamikkum*, or 'concave square'. This is the figure formed by subtracting from a square pieces determined by four circles of equal radius and tangent at the corners of the square. We will discuss some of the occurrences of this figure and the problems that were set using it.

Who does mathematics?
Madeline Muntersbjorn, University of Toledo

The mathematical community is both inclusive and exclusive. On the one hand, Plato suggests that everyone, even slave-boys, can do math. Since Plato, the realm of mathematicians has grown to include not just young and under-privileged men, but women, animals, aliens and machines. Cognitive scientists suggest that certain birds and primates can do arithmetic. The search for extra-terrestrial intelligence (SETI) operates under the assumption that alien intelligence, if it exists, could interpret signals broadcast in the universal language of mathematics. Once upon a time, scores of women toiled to find results we now generate using hand-held calculators. Today, desktop computers prove theorems we suspect are true but cannot demonstrate on our own. On the other hand, mathematics is a highly specialized discipline. Plato's inscription, "Let no one enter who has not studied geometry," is an exclusionary injunction against the uninitiated. Professional mathematicians speak incomprehensible languages and inhabit unimaginable worlds of unlimited dimensions and impossible objects. Mathematics teachers struggle in vain against the all-too-common refrain, "I'm just not a math person." In this paper, I survey the borders demarcating mathematicians from other kinds of beings. These borders are elusive and refuse to stay in any one place for very long. Their instability tempts us to regard "mathematician" as a label with multiple meanings. However, there is continuity in the mathematical community and unity in the multiplicity of mathematicians, not in spite of dynamic increases in the scope of mathematical reasoning, but because of them.

Reciprocal Trajectories and Euler's Marvelous Geometry Machine
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A curve is a reciprocal trajectory if it has an axis such that if the curve is reflected across that axis and then translated along the axis any distance, then the resulting curve intersects the original curve at a right angle. The problem of finding reciprocal trajectories, particularly algebraic ones, was a hot topic in the early 18th Century, though the problem is almost entirely forgotten today. We review Euler's solution to the problem, and the marvelous way he integrates calculus and geometry into the construction that gives his solution.

Curry's Anticipation of the Types Used in Programming Languages

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The information stored in a given memory location of a computer is a string consisting of the digits 0 and 1. These strings can be interpreted in various ways, and types are used in programming languages to indicate the interpretation. The earliest higher level languages had only simple types, such as integer, long integer, real, boolean, etc. But more advanced languages have a need for the types of functions. In the functional languages in particular, such as LISP and ML, types are first class objects, and require types of the form A -> B for a function that takes arguments of type A and has values of type B.

One of the standard models of types in computer languages is type assignment in lambda-calculus. This, in turn, is often traced back to a paper by Alonzo Church [1]. But it can also be traced back to work by H. B. Curry which began in the late 1920s. His first paper on this, [3], was actually written in 1932-33 (and he published [2] because it took so long to publish [3]). This paper was, in many ways, an anticipation of [1], although that was not really clear until the publication of [4, Chapter 9].

An entry in Curry's notes for March 29, 1956, introduces what is now called the "dependent function type", a type for a function which has the property that the type of the value of a function may depend on the argument as well as the type of the argument. Dependent function types have become important since the 1970s.

This talk will trace Curry's work on this subject throughout his career.

References

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George Airy: The Conservative Lucasian

Jim Tattersall, Providence College

We discuss the life and mathematical accomplishments of Sir George Biddle Airy, K.C.B., M.A., LL.D., D.C.L., F.R.S., F.R.A.S. Airy, a Trinity College, Cambridge graduate, Senior Wrangler, First Smith's Prizeman, Plumian Professor, Astronomer Royal, President of the Royal Society, served as the Tenth Lucasian Professor of Mathematics at Cambridge University from 1826-1828.

Visual Reasoning in Mathematical Proof: From Bolzano to Cantor and Beyond

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Mathematicians commonly use pictures and diagrams to explain mathematical concepts to students. In his seminal work on the process of discovery in mathematics How to Solve It, Georg Polya advises using pictures to make problems more concrete. Then students can proceed to the more abstract business of constructing a formal proof. Few philosophers, however, take seriously the idea that a picture can have genuine epistemic value as a species of proof in itself

In this paper I examine two historical cases of pictures or diagrams in mathematical reasoning: 1) the Intermediate Value Theorem, proven by Bolzano; and 2) Cantor's diagonal argument for the countability of the rationals. My analysis of these contrasting cases of picture proofs suggests that it is possible to formulate a more general account of what makes a picture count as an acceptable form of mathematical proof. Good picture proofs, I argue, contain clear visual correlates of formal axiomatizable mathematical notions; finding connections between these modes of mathematical expression may well expand the range of methods we can use to do mathematics.

In the course of discussion of the two cases, I will present two differing views on the status of picture proofs. Marcus Giaquinto considers picture proofs suggestive of formal proofs in that they "bring to mind a form of non-visual thinking". He also notes that pictures can be misleading in fields of mathematics like analysis where the key concepts do not have adequate visual correlates. The Bolzano case is a key example of this problem: the unbroken curve in the drawing demonstrates both the continuity and differentiability of the function depicted, and the visual representation conflates the two mathematical notions. James R. Brown disagrees with this account. He argues for an inductivist model of evidence for pictures in mathematics. Contra Giaquinto, he says that pictures provide the 'known to be true consequences that we use for testing the hypothesis of arithmetization. Trying to get along without them would be like trying to do theoretical physics without the benefit of experiments to test conjectures." For Brown, the Bolzano case is a successful picture proof because it trades on one notion of continuity—the so-called "pencil continuity—while the formal proof trades on the formal ε-δ definition. He claims that "the fact that the picture is convincing is explainable only by assuming that pencil continuity and ε-δ continuity are related."

My position on the Bolzano and the Cantor cases (I discuss Giaquinto and Brown's accounts of this case in brief) differs strongly from Brown's and somewhat from Giaquinto's. I argue against Brown's inductivist model and suggest an account that places picture proofs within the realm of deductive mathematical procedures. I extend Giaquinto's insights as a way to help provide more formal criteria for acceptability of picture proofs. The general idea is this: picture proofs encode or contain information from which one can construct a formal proof. Creating levels of formality of mathematical reasoning (from pictures to stories to informal proof sketches to formal proof, with intermediate steps along the way) shows the continuity between the context of discovery and the context of justification in mathematics.