

# CSIIPM 1996 Annual Meeting

## Brock University

### St. Catharines, Ontario.

#### May 30 - June 1, 1996

#### Program and Abstracts

No 2nd projector for  
Krug's  
Blackboard for Jones

Thursday, 30 May

9:00 - 9:30: Peter L. Griffiths, 'Fermat's Last Theorem'

Fermat's theorem that there is no integer solution to  $a^n+b^n=c^n$  (with  $n>2$ ) has not hitherto been clearly and convincingly proved (except by hearsay) because mathematicians have focused attention on the various values for  $n$  instead of on the relationships between  $a$ ,  $b$ , and  $c$ . They have also failed to distinguish between nil possibility and infinitesimal possibility. A mistake has been made of initially assuming  $c$  to be an integer, when the whole question concerns the probabilities of  $c$  being an integer when  $a$ ,  $b$ , and  $n$  (2) are integers.

Mathematicians have triumphantly inserted and removed all common factors from  $a$ ,  $b$  and  $c$ , when all that is required is to remove the common factors from  $a$  and  $b$ , so that at least initially  $c$  is not an integer. An important but little noticed special case of this arises when  $a=b$ . The general conclusion is that the correctness or incorrectness of Fermat's Theorem depends on three conditions: 1. If  $a^n$  is low (for example 1) then  $a^n+b^n$  cannot equal  $c^n$  so under this condition Fermat's Theorem is correct. 2. If  $a^n$  increases to just below  $b^n$  then there is an infinitesimal possibility that  $a^n+b^n=c^n$ , so that under this condition there is an infinitesimal possibility that Fermat's Theorem could be wrong. 3. If  $a^n=b^n$ , then Fermat's condition will be correct if  $n$  is finite, but is incorrect if  $n$  is infinite. Fermat's Theorem is therefore mostly correct unless  $n$  the power is infinite, and  $a=b$ . The equality of  $a$  and  $b$  has a considerable effect on the integer relationship of  $a$  and  $b$  to  $c$ .

Ref. of irrational

Addressing social class

Language established first (there may be examples to the contrary)

9:30 - 10:00: Elaine Howes and Bill Rosenthal, Less than Zero

For the past two years, we have been engaged in a study of conceptions, constructions, and constrictions of mathematical infinity. Beginning with our own fascinations with and fears of the infinite, we first set out to develop from a feminist poststructuralist perspective a re-vision of the abstract, eerily disembodied discipline of Mathematics, utilizing the experiences, perceptions, and critical faculties of one who has been successful scaling its slopes (Bill) and one who has chosen to avoid the climb (Elaine). We have explored the canonical contrivances of infinity; intersected and contrasted mathematicians' tamings of the infinite with our (inter- and intra-) personal ideations, senses, and sensibilities; disinterred infinities that have been marginalized, forgotten, and scorned by (100-epsilon)% of Mathematical historians, philosophers, historiographers, anthropologists and sociologists,

Infinity source of boundless problems

Participatory: groups of 3 to talk about origin of infinity

psychologists, and cultural critics. Our research findings: defy and comprehensible summary, much less a comprehensive listing. Nevertheless, we offer some clips from the highlight film. \* Contrary to unanimous historical consensus, mathematical infinity didn't begin with Zeno. \* There exists a plethoric panoply of conceptualizations of infinity -- Mathematical, poetic, and personal - neglected by the lion's share of philosophical and historical scholarship, as well as the popular accounts dependent on them. \* The roots of the mathematical infinite (a) grew in the same soil from which sprang the dichotomies that soon became the canonical basis for Western thought, particularly and especially the subordinations of the body to the mind and the feminine to the masculine; (b) are correlated with and possibly causally related to the suppression of paganism and the development of monotheistic male God worship. \* We submit that the discernible fear of the infinite running through mathematics maps onto the woman-hating and terror of women's 'uncontrollable' and 'omnivorous' sexuality so evident in the post-Socratic social order.

10:00 - 10:30: Tracy A. Glenn, Local Mathematics

I argue that universal truths in mathematics and physics aren't just expressions subsuming a variety of interpretations and concrete applications, but rather what is often taken as a single law or a single theory really represents a constellation of slightly different models with terms modified, added or dropped out to fit the local context. I support the position taken by Joseph Rouse in his essay "Local Knowledge," that what is thought to be a process of abstraction or extraction of some essential truth from nature is actually a process of making a series of tradeoffs among the demands and constraints of local conditions. While Rouse spoke generally about theories in physics, I extend his arguments to show that even mathematical theories are shaped by local practices and physical circumstances, and so cannot be considered to be decontextualized knowledge. Various traditions, norms and standards specific to a particular branch of mathematics shape a theory just as much as physical circumstances do in more concrete disciplines. Requirements that a theory be formally proved or that it be axiomatized may result in assumptions and modifications that move it farther away from both universality and truth. Adherence to such norms in mathematics then often contributes to the creation of esoteric local knowledge rather than universal truths. It is more accurate to say that there is a tradeoff between different types of standards and rigor involved in formulating and 'abstract' theory. While formal demands increase when a theory is moved into the context of pure mathematics, other more empirical demands may be relaxed. Thus, theories are neither 'decontextualized' nor necessarily made more universal when they are appropriated by mathematicians.

Sociology of math (unmarked) it's eye contact (eyes shut?)

10:30 - 11:00: Darcy Cutler, Completeness and Logic

It is sometimes argued that the semantic incompleteness of standard second-order logic is evidence that standard second-order logic is not really logic.

This kind of argument must rely on a prior claim that the notion of logical

ca. 1/90's  
Foucault's notion of discipline  
This kind of argument must rely on a prior claim that the notion of logical foundation is not built on.

consequence is only adequately modelled by a formal semantics that can be coupled with a system of deduction that is complete with respect to it. In *Foundations Without Foundationalism: A Case for Second Order Logic*, Stewart Shapiro argues that the only kind of argument that would support such a claim rests on a "foundationalist" view of logic. According to this view, logic is devised in order to provide an incorrigible epistemic foundation for mathematics. Shapiro argues that mathematics as it is practised informally is "as certain as it needs to be" and requires no foundationalist reconstruction. Once we abandon "foundationalism" we are free to embrace the view that second-order semantic consequence is a reasonable model of informal logical consequence. I join Shapiro in rejecting foundationalism. Nevertheless, I argue that completeness is a desirable property for a logic to have. One of the things we require of a logical principle is that it be "topic neutral". Logical principles are to be common to all fields of knowledge. If a conclusion  $c$  is a logical consequence of a set of premises  $P$  then the fact that  $c$  is a logical consequence of  $P$  depends on no principle from any particular field of knowledge. If a notion of semantic consequence cannot be associated with a complete system of derivation then in general, we can only know that the relation of consequence holds by resorting to semantic principles which belong to a specialized field of mathematical knowledge, i.e. model theory or set theory. So if we can only know that  $c$  is a semantic consequence of  $P$  (in a particular semantic theory) by means of semantic reasoning, we have no guarantee that  $c$  is a logical consequence of  $P$ . Hence only a semantic theory that can be coupled with a complete system of deduction can be a reasonable model of logical consequence.

11:00 - 11:30: Hardy Grant, Some Thoughts on the History of Beauty in Mathematics

"Euclid alone", says a modern poet, "has looked on beauty bare." But - one may ask - did Euclid think so? Did anyone else in ancient Greece? I shall try to identify the things that ancient writers took to be beautiful, and to compare their sentiments with the corresponding feelings attested by creative mathematicians in our time. It will appear from this contrast that the rise, since the Renaissance, of modern mathematics and of mathematized science contributed a new dimension to western aesthetic sensibility. One theoretician of beauty saw this with particular clarity, and I shall briefly expound his views.

11:30 - 12:00: Israel Kleiner, A historically Focused Course in Abstract Algebra

12 - 1:30 Lunch Break and Council Meeting

4:30 F

1:30 - 2:00: Richard O'Lander, The History of the New Math

Today there is a great outcry from parents, teachers, business people, politicians and others about the low academic performance of many of our high school graduates. This is especially true when it comes to mathematics. "How will we compete with the Japanese and Germans?" is the rallying cry. This is not the first time such a concern has been raised. The 1950's and 1960's were a time of great curriculum reform in the pre-college mathematics. The reform movement was initiated in part by the supposed "technological gap" between the United States and the Soviet Union. The purpose of this paper is to discuss the basic premise behind the "New Math", as well as its successes and failures.

Public

2:00 - 2:30: Erwin Kreyszig, Leonhard Euler (1707-1783) as Applied Mathematician and Engineer

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Leonhard Euler is the most prominent mathematician of his century (perhaps besides Lagrange). His time is characterized by a rapid extension of the calculus into vast uncharted mathematical territories. The impact of Euler's work in pure mathematics was crucial to the development of mathematics in Central and Western Europe. Gauss, Lagrange, Laplace, and other leading mathematicians of the next generations based much of their work on Euler's accomplishments. Many of our present notions and notations, for instance in trigonometry, are Euler's creations.

This talk will concern some of Euler's fundamental contributions to applied mathematics, whose details are less known and sometimes not easy to locate within the eighty-five volumes of his Works, some of which are still to appear. This will include a study of Euler's path-breaking work in hydrodynamics, with engineering applications to the construction of sailing ships and water turbines. To round out the presentation, we shall also mention some less known facts from Euler's life.

2:45 - 3:30: Gregory H. Moore, Cantor, Hausdorff and the Emergence of Order, 1885-1908

The general concept of an 'order' was among the very first classes of abstract structures to be explicitly introduced. In an unpublished paper of 1885, and one published in 1887, Cantor introduced the notions of order, order-isomorphism, and order-type. (Among the classes of abstract structures, only the class of finite groups, introduced by Cayley in 1854 and reintroduced in 1878, was earlier. The class of fields appeared a bit later, in 1893.) But Cantor did not get very far with the general notion of order - in contrast with the deep results that he obtained on well-ordered sets and their ordinals. Cantor succeeded in characterizing the order-types of the natural numbers, the rationals, and the reals with their usual order. But his primary interest in order-types (beyond the well-ordered sets) was in using  $n$ -fold ordered sets to generalize the topology of  $n$ -dimensional Euclidean space, by means of closed sets, perfect sets, etc. Hausdorff began investigating order-types in 1900. His most ground-breaking contributions on order-types came during

the years 1906-1908, when he introduced the notions of cofinality (central to later work on order-types, ordinals, and cardinals), singular and regular cardinal, and inaccessible cardinal. At this time he found various results on order-types that are equivalent to the Continuum Hypothesis (CH), and he introduced the Generalized Continuum Hypothesis (GCH). But he ran into the difficulty that neither GCH nor CH could be proved, and hence many central questions about order-types remained undecided. No deeper understanding of this matter was reached until the seminal work of Goedel (1938) and Cohen (1963) on models of set theory.

### 3:30 - 4:00: Rebecca Adams, From Analysis to General Topology via the Borel Theorem

Borel's Theorem (1894) was given for a linear space. Fréchet introduced the term compact, which he used to prove the Borel Theorem in an abstract space (1906). Hausdorff essentially showed the equivalence of Frechet's compactness and the Borel-Lebesgue theorem in metric spaces (1914). Alexandrov and Urysohn defined (bi)compactness as the Borel-Lebesgue property for topological spaces (1924). Considering the extension of this theorem to abstract spaces (1904-1924) offers an intuitive appreciation of the transition from real analysis to general topology. Work by Alexandrov, Urysohn, Chittenden, Frechet, Hausdorff, Hedrick, Kuratowski, Sierpiński, and R. L. Moore is included.

### 4:00 - 4:30: R. Godard, The Process of Axiomatisation in the Theory of Probabilities

In 1933, Kolmogorov in Russia published The Foundations of the Theory of Probabilities. We quote: "Theory of Probabilities, as a mathematical discipline, can and should be developed from axioms in exactly the same way as Geometry and Algebra... However, if our aim is to achieve the utmost simplicity both in the system of axioms and in the further development of the theory, the postulational concepts of a random event and its probability seem the most suitable." We have tried to trace the evolution of the Theory of Probabilities from the end of the XIXth century up to Kolmogorov, and the theory of distributions. On one hand, we study the contribution of Bernstein, Borel, Cantelli, Copeland, Frechet, Lomnicki, Reichenbach, Slatsky, Steinhaus, Tornier. In particular, we follow the introduction of Lebesgue's theories of measure and integration. On the other hand, we try to do justice to the school of mathematical logic with deMorgan who published its Theory of Probabilities in 1845 and John Venn who published The Logic of Chance in 1866.

### 4:30 - 5:00: Kurt Ramskov, The emergence of mathematical institutes

One thing which separates mathematics from the natural sciences is that no equipment is necessary to do mathematical research (except for literature, pencil and paper). However, today we naturally associate with a

mathematical department the existence of a mathematical institute, i.e. a building including offices to mathematical professors and students, a mathematics library, lecture rooms, etc. This talk will describe the emergence of the first mathematical institute in Copenhagen 1929-34 and relate it to some of its predecessors. Questions to be discussed include: - Which arguments were used to argue for the building of mathematical institutes? - How were the previous institutional conditions for mathematics? - Why was the majority of mathematical institutes constructed from the 1920's and onward?

### Friday, 31 May

### 9:00-9:30: Ed Cohen, Gregorian Dates for the Jewish New Year

*Other New Years - Venerable Bede*

Because of its luni-solar nature, the Hebrew calendar, it seems, moves eccentrically. The beginning, called the Jewish New Year (Tishrei 1), has its appearance in the 19th and 20th centuries anywhere from September 5 to October 5. Many have written on this subject, some giving tables, some giving expositions or proofs. The purpose here is to present a history on this topic for the currently used Hebrew calendar. Gauss and the journals of Baron von Zach play an important role in this investigation.

*Quantitative conspiracy of calendar the Jews*

### 9:30 - 10:00: Dominique Vellard, Nondecimal numeration systems in nonliterate societies

### 10:00 - 10:30: James Tattersall, Davenant's Problem

*Talks course for Joseph Raphel*

Edward Davenant (1596-1680), Fellow of Queen's College, Cambridge, instructed the diarist John Aubry in algebra and corresponded regularly with John Wallis and Archbishop Ussher. Christopher Wren considered Davenant the greatest English mathematician of the early seventeenth century. Davenant retired from academic life at age 30 upon receiving a living at Gillingham in Dorset. Being wary of what his parishioners would think of what he did in his spare time he published very little of his mathematical work. In 1675 he communicated an interesting algebraic problem to Thomas Baker, author of the Geometrical Key (1684) who disseminated the problem. Besides Baker solutions were obtained by both Collins and Newton.

### 10:30 - 11:00: Craig Fraser, Hamilton-Jacobi Mechanics and the Development of Weierstrassian Field Theory in the Calculus of Variations

A well known change occurred in the relationship between mathematics and physics in the 19th century. In the earlier period there were rudimentary links connecting analysis and theoretical mechanics. D'Alembert for example regarded mechanics as a branch of mathematics, a conception that he developed in some detail in his Traite de Dynamique of 1743. Lagrange believed that he was reducing mechanics to analysis through the algebraic

*No show*

programme set forth in his *Mécanique Analytique* of 1788. It was not a question here of applying mathematics to mechanics: mechanics rather was regarded as part of mathematics. The 19th century by contrast witnessed the development of theoretical physics on the one hand, and an ideology of pure mathematics as distinct from applied conceptions on the other. The calculus of variations was a part of mathematics that was rooted historically in problems in geometry and mathematical mechanics. During the second half of the 19th century however it like other parts of analysis was developed systematically along formal, logical lines. Functional concepts (weak and strong extrema), questions of existence (implicit function theorems), methodology (distinction between necessary and sufficient conditions, rigour) were prominent features of Weierstrass's famous lectures of the 1870's on the calculus of variations. By 1900 the calculus of variations had become thoroughly grounded as a branch of pure, modern abstract analysis. At the end of the century Hamilton-Jacobi methods originating in mechanics served as a fundamental source for new ideas in the purely mathematical subject of Weierstrassian field theory. Introduced in the writings of Beltrami and Kneser these ideas led in 1900 to Hilbert's invention of the invariant integral. This was a development that Weierstrass himself had failed to anticipate. It constituted a remarkable and rather unexpected influx of ideas from an applied source into the domain of pure analysis.

#### 11:00 - 11:30: Francine Abeles, Infinitesimals are Numbers

The idea that a number system can include infinitesimal and infinite numbers belongs to Gottfried Leibniz; its realization was constructed by Abraham Robinson some 300 years later. This extended number system has, among its many applications, hyperbolic geometry which is based numerically on non-Archimedean fields. In an obscure book published in 1888, Charles L. Dodgson presents geometrical arguments relating the non-Archimedean property with the ordering of infinitesimals in which he foreshadows Robinson's notion of standard numbers. In this talk I will sketch the main historical points with particular emphasis on the background to Dodgson's work.

#### 11:30 - 12:00: Christopher Baltus, Separating roots of a polynomial: Lagrange and his successors

It was Lagrange who created "the theory of equations" as a distinct and coherent subject. His numerical work on equations was presented in a book, *Traité de la Résolution des Equations Numeriques...* (1798, 2nd ed 1808), incorporating two earlier papers and with added "notes" occupying the larger part of the book. Of pivotal importance in equation solving is the "separation of roots": the discovery of disjoint intervals, each with one root and which, together, contain all the (real) roots. Lagrange offered a couple methods. Budan, Fourier, and Sturm developed another approach. After a long hiatus, Lagrange's approach has reappeared in recent work, particularly that of a computer scientist. A. G. Akritas.

#### 12 - 2:00 Buffet Lunch and Annual Meeting, Trillium Room

#### 2:00 - 2:30: Evelyne Barbin, L'ordre d'invention dans les mathématiques et dans la philosophie de Descartes

Dans tous ses écrits, Descartes présente les mathématiques comme un modèle de certitude et de fécondité, et comme une exercice propédeutique pour la pratique de la méthode. Mais les mathématiques auxquelles il pense ne sont pas les mathématiques "ordinaires". En effet, dans *La Géométrie* de 1637, Descartes propose une nouvelle conception des objets et de l'objet de la géométrie, et de la démonstration mathématique, selon un ordre d'invention qui décompose et recompose les figures de la géométrie en objets simples. Descartes oppose cet ordre d'invention, qui s'adresse à l'intelligence du lecteur en reposant sur l'évidence des objets simples et des déductions, à l'ordre axiomatique-déductif des *Elements* de géométrie "ordinaires", qui convainc "en arrachant le consentement du lecteur".

#### 2:30 - 3:00: Katherine Hill, 'Juglers or Schollers?': The Role of Instruments in Mathematical Education

The relationship between theory and practice in mathematics, and consequently the proper role of mathematical instruments, was a subject of intense debate in early modern England. For example, although William Oughtred invented several instruments, including a horizontal instrument and a double horizontal dial, he discouraged teaching the use of instruments to beginning students of mathematics. Instead, he advocated postponing the use of mathematical instruments until after the theoretical foundations of a subject had been thoroughly mastered. Oughtred, however, never provided a systematic exposition of his views on teaching mathematics. But several of the publications surrounding his priority dispute with Richard Delamain regarding the Horizontal instrument, a device that graphically determined solutions to problems concerning the position of the sun, delineate the differences between Oughtred's views and the opinions of a group he labelled 'vulgar teachers.' These 'vulgar teachers' were accused of ignoring theory in favour of practice; they concentrated the applications of instruments 'to make their Schollers onely does of tricks, and as it were juglers.' Instruments, Oughtred claimed, could only be used with understanding by students who had a proper theoretical foundation. This paper will explore the difference of opinions between the 'vulgar teachers' such as Delamain and the more academic, or pure, mathematicians such as Oughtred on the proper mixture of theory and practice and the proposed role of instruments in mathematical education. Moreover, it will also examine the possible motives behind the opposing educational methodologies.

#### 3:00 - 3:30: John D. Anderson, Some Pearls of Geometry

When mid-17th century mathematicians were groping to understand the significance of the new powerful analytical methods of "Cartesian geometry," the re-examination and extension of the traditional canon of curves played a

vital role in their investigations. Some scholars introduced new curves styled on traditional geometric symptomata which were closely related to ancient locus problems. During 1657 and 1658, Rene Francois de Sluse corresponded with Christiaan Huygens concerning a new variety of curves he had developed. These "pearls" of Sluse were his most memorable addition to the collection of curves studied by mathematicians. Also, his work on the cissoid marks a transition from classical curve definition by considering, for the first time, the portion of the curve outside the circle as it is used in the classical Greek construction of the cissoid. In addition, Sluse briefly studies a new kind of "ellipse," which he recognized as being the first of the spiric sections of Perseus. In the years preceding Frans van Schooten's second Latin edition of Descartes' *Geometrie* there had been a great deal of work done on the methods of "Cartesian geometry," much of which van Schooten included as commentaries to this new 1659 edition. Sluse's work is not found among them. Historians of mathematics have traditionally condemned his misguided understanding of the applications of coordinates to the study of curves. I argue that Sluse's work represents, rather, an important middle ground between classical geometrical methods and the fast developing "Cartesian geometry." Moreover, his adept use of classical techniques illustrates that coordinate concepts and methods were neither clearly understood nor were even applied preferentially (or even consistently) to geometric problems.

**3:30 - 4:00: Hardy Grant, Israel Kleiner, and Abe Shenitzer, Some significant developments and turning points in the history of mathematics**

This talk will consist of brief descriptions of important developments in the history of mathematics bearing on some of the following topics: 1. The emergence of general problems and general methods in the 17th century. 2. Beyond three dimensions. 3. From arithmetic to arithmetics. 4. Some aspects of the evolution of algebra. 5. Some mutations of the curve concept. 6. Some aspects of the issue of rigor from Archimedes to Weierstrass and beyond.

**4:00 - 4:30: Barnabas Hughes, Early Voyages into Logarithmic Seas**

**Saturday, 1 June**

**8:00 - 8:30: Ronald Sklar, The Use of Logic in Automated Theorem Proving: A Historical Sketch**

The idea of mechanizing mathematics can be traced back to Descartes and Leibniz in the 17th century. But the first truly automated proofs in mathematics had to wait until the 1950's and the invention of the electronic computer. The purpose of this talk is to trace the use of logic in automated deduction with particular emphasis on the use of the principle of resolution. Along the way the contributions and ideas of Frege, Peano, Skolem,

Herbrand, Hilbert and Ackerman, Goedel, Church, Turing, Davis and Putnam, J. A. Robinson, Wos and others will be briefly discussed.

**8:30 - 9:00: Thomas L. Bartlow, Kenneth O. May and the Theory of Social Choices**

Shortly after completing his doctoral work in 1947 Kenneth May investigated the probability that one party will win a majority of popular votes while another wins a majority of the districts. Following the appearance of Kenneth Arrow's *Social Choice and Individual Values* in 1951 May wrote on conditions for majority decision and on the role of intransitivity in individual and group decisions. This research was done during a time when May was becoming less active in mathematical economics and more involved in issues of mathematics education, some years prior to his interest in the history of mathematics.

**Special Session: (June 1)**

**9:00 - 10:00: Alexander Jones (Invited one-hour talk), Greek Applied Mathematics**

Mathematical methods and reasoning manifested themselves in classical antiquity beyond the confines of the familiar geometrical literature. The applied side of Greek mathematics has become the focus of much of the most interesting current historical research, as long-neglected texts are reexamined and new documents come to light. Problems originating in physical science, technology, and astronomy can now be seen as the inspiration of good mathematical work well after the so-called Golden Age.

**10:00 - 10:30: Daryn Lehoux, The Locus Theorem in Pappus and Proclus**

Both Pappus and Proclus offer discussions of the definition and classification of locus theorems. A close comparison of their treatments reveals that there are problems with Proclus' account, both in his classification and his definition. While the discrepancies between the two authors' classifications of loci are not on the whole insurmountable, it is shown that Proclus has some difficulty with the idea of solid loci, and that he exhibits some confusion on the question of how to treat the cylindrical helix. The definitions of loci in Pappus and Proclus, on the other hand, differ markedly. Through an attempt to work out their respective definitions in full detail, it is found that the inconsistencies are not limited to discrepancies between the two authors, but that Proclus' definition and description of locus theorems (in particular that at 'In Primum Euclidis' I.35) is in fact self-contradictory. Furthermore, since Pappus' discussion is both self-consistent and, to some extent, corroborated by Eutocius, we must conclude that Pappus had a thorough grasp of the subject, and that Proclus, who was a philosopher rather than a working mathematician, was struggling with the difficult problem of loci and did not completely understand it. This fact will have to be taken into account in any attempt to understand the ancient idea of loci.

10:30 - 11:00: Coffee Break

11:00 - 11:30: W. S. Anglin, Did Zhao Shuang Prove the Theorem of Pythagoras?

Zhao Shuang (250 AD) was the first Chinese mathematician, as far as we can tell, who had a proof of the theorem of Pythagoras. Claims by J. Needham and others that such a proof is found in an earlier Chinese text (c. 100 AD) cannot be substantiated. We reach these conclusions through an examination of the original texts, and a critique of a translation by B. Gillon. We append a translation of some comments of Zhao Shuang which are relevant to our topic, but which have never before been translated into any European language. This translation was done with the help of Yu Jiyuan and Grace Zhang.

11:30 - 12:00: Joran Friberg, From Susa to Syracuse. Square roots and square root approximations in the ancient mathematical tradition

The study of Mesopotamian mathematics has lately been revolutionized through dramatic reinterpretations of known cuneiform texts, and through the publication of many new cuneiform mathematical texts, from proto-Sumerian to Late Babylonian. In the process, it has become more and more obvious how much Greek mathematicians may have been inspired, via intermediaries, by their Babylonian predecessors. In the present talk, this thesis will be defended by examples fetched from the early history of square root approximations. These include Old Babylonian mathematical texts from Susa, a new Late Babylonian text on (among other matters) the area of an equilateral triangle and roughly contemporaneous material from the demotic Papyrus Cairo.

The method used by Heron for approximation of square roots, mentioned in *Metrica I*, 8, is essentially the same as the Old Babylonian method. This method is used in nearly 50 examples in Heron's works. In a handful of other examples (including #883), a different method is used, which can be shown to rely on "composition" of triangle sides. (A related composition method is used in one of the Old Babylonian texts from Susa.) This method was probably used also by Archimedes in order to find the very accurate estimates  $265/153$

12:00 - 1:30: Lunch

1:30 - 2:00: Samuel Kutler, What Did Euclid Hope to Accomplish with his 'Elements'?

By concentrating on the order of Euclid's 13 books, the order of the propositions in certain books, the terms that are and are not defined, the

principles from which the propositions are deduced, and the title itself, I shall attempt to glean the aims that Euclid must have had in mind in framing his *Elements*.

2:00 - 2:30: Jonathan P. Seldin, Two Remarks on Ancient Greek Geometry

1. Although physicists tell us that the universe is probably not Euclidean, Euclidean geometry is still the standard to which all other geometries are compared. Why is this the case? Why is this the geometry that the ancient Greeks developed? Some modern results suggest that our biological programming may be responsible for making Euclidean geometry the first geometry found by any human culture that takes up geometry in a systematic way. 2. We have become used to the idea that our real number system gives us what the ancient Greeks called "magnitudes". However, large parts of geometry can be carried out using smaller fields. For example, all ruler and compass constructions can be carried out if magnitudes are taken to be the surd field, and this fact is used to show that certain constructions cannot be carried out by ruler and compass. The ancient Greeks did use some constructions that were not ruler and compass, so that the surd field is too limited to serve as the magnitudes for all of ancient Greek geometry. But it is still worth asking whether all the real numbers are really needed to reconstruct what the ancient Greeks did. (Since the ancient Greeks viewed the magnitudes as being given rather than constructed, they would probably not have understood the point of this question.) This part of the talk will explore the idea of using a countable field instead of the real numbers for the ancient Greek magnitudes. The field in question is obtained from the algebraic numbers by adding  $e$  and  $\pi$  and then closing under  $\exp$ ,  $\ln$ , the trig functions, and the taking of square roots.

2:30 - 3:00: J. L. Berggren, Mathematical Aspects of Ptolemy's 'Geography'

In this talk we shall examine mathematical methods used or implied in Ptolemy's *Geography*, such as minimizing cartographic distortions, applying theorems of spherical geometry, solving simple triangles and using chord tables.

3:00 - 3:30: Coffee Break

3:30 - 4:00: Whitney Johnson and Bill Rosenthal, The Reflection of Early Greek Mathematics in the Mathematics of Today

Many are quick to anoint the ancient Greeks, most notably Pythagoras, Aristotle, and Plato, as the founders of modern mathematics. Many also feel indebted to pay homage to Euclid for his eloquent unification of the works of the mathematicians who preceded him. Just as there were many who contributed to Euclid's work, so too Pythagoras, Aristotle, and Plato stood on the shoulders of the giants who preceded them. When we carefully study

their philosophies, we find the thoughts of the pre-Pythagorean philosophers, particularly Hesiod and his scions Anaximander, Anaximenes, and Xenophanes. The primary concern of these men was not limited to the distinct and distinguished spheres that moderns demarcate as mathematics and the sciences. Inestimably more catholic in their worldview than we, the pre-Pythagorean philosophical domain was cosmological, its Holy Grail nothing less than the original source of the universe. They questioned their surroundings not only empirically and analytically, but also in a spiritual sense, posing and answering enquiries integrating creation myths, the character of the divine, the nature of the infinite, and epistemology. Although the pre-Pythagoreans were not focussed on the development of mathematics as we know it, one can find and we have found reflected and refracted traces, akin to the cosmic background radiation bequeathed to us by the Big Bang, of the questions, ideas, and interests that populate the canons and controversies of modern mathematics.

#### 4:00 - 4:30: Glen Van Brummelen, Use and Abuse of Statistics in Ancient Astronomy

The powerful tools of analysis placed in our hands by the advent of statistics in this century are transforming the methods by which we interpret our culture --- not least in the history of science. This power is, however, offset by the ease with which statistics can be misused. In the history of astronomy, statistics has been brought to bear primarily by those opposed to conventional views on the subject. As a result, the insights gained through its use have not been studied critically; and its methodologies, advantages, and dangers are not well understood. We will survey the history of statistics in the history of astronomy, concentrating on (but not limited to) Ptolemaic studies. We will elucidate the nature and drawbacks of statistical reasoning, and attempt to define the proper role of statistics in the history of science.

### Category theory and the foundations of mathematics

#### Joint session with CSHPS

June 1, 2:00 to 5:00 PM.

#### J. Lambek, Categories in Foundations

The objects of a category, like the monads of Leibniz, have no windows. To infer what is inside an object  $A$  one need only look at all arrows into  $A$ . While the category of categories had at one time been proposed as a foundation of mathematics by Bill Lawvere, categories appear more readily as deductive systems: deductions  $A \rightarrow B$  may be viewed as arrows in a category, provided one pays proper attention to the relation of equality between such arrows. Thus, for example, the positive intuitionistic propositional calculus may be viewed as a cartesian closed category in the sense of Lawvere, who also studied cartesian closed categories equipped with a subobject classifier and a natural numbers object. He realized that, in such an elementary topos, one may interpret the language of mathematics, say

when represented as type theory. Of special interest are local toposes, which have the disjunction and existence properties. They serve as models in the sense of Goedel and Henkin, even when the type theory is allowed to be intuitionistic. Goedel's completeness theorem then says that a closed formula of the language will be a theorem if it is true in all models, while the incompleteness theorem asserts that it does not suffice to consider only models with the following property: if  $\phi(n)$  is true in the model for all standard numerals  $n$ , then  $\forall x \in \mathbb{N}, \phi(x)$  is true in the model. Surprisingly, for pure intuitionistic type theory, a single model suffices, the so-called free topos, which may be proposed as a distinguished world of mathematics acceptable to modern formalists, Platonists, and intuitionists.

#### John Bell, Remarks on Category Theory

I shall offer some observations on category theory as an instrument for representing mathematical form, contrasting its efficacy in this regard with set theory.

#### Colin McLarty, Category Theory as the Science of Resnik's "Structures"

I explicate Resnik's "patterns" (or "structures") as categories. On the one hand we can then apply Resnik's philosophical analysis to categorical foundations. On the other hand, it shows how familiar tools from categorical practice can solve open questions about Resnik's views - especially the question of how a mathematical object can at once be a "featureless point" of a pattern and a pattern itself.

#### J. P. Marquis, The Ontological Status of Category Theory: The Case of the Adjoint Functors

As Eilenberg and MacLane claimed in their 1945 paper introducing categories, category theory can be thought of as an extension of Klein's Erlanger program. It is in this spirit that we will consider in this paper what we take to be one of the important and mysterious aspects of the contemporary mathematics brought to the fore by the introduction of categorical concepts: the abundant existence of adjoint functors. We will thus - at least at first - sidestep the general philosophical question of the existence of categories, which from the point of view we are placing ourselves would be analogous to the question of the existence of groups in the context of classical geometry, and focus on the more specific question: why are adjoints everywhere? We will try to survey the possible explanations underlying this fact and explore their ramifications.

#### William Anglin: Commentary