

**Canadian Mathematical Society Winter Meeting  
December 6-8, 2003  
Simon Fraser University, Vancouver**

History of Mathematics  
Org: Len Berggren (Simon Fraser)

Sunday December 7

- 16:00 - 16:30 Israel Kleiner, HC-1700
- 16:30 - 17:00 Tom Archibald, HC-1700
- 17:00 - 17:30 Gregory Moore, HC-1700
- 17:30 - 18:00 Ram Murty, HC-1700
- 18:00 - 18:30 Lawrence D'Antonio, HC-1700

Monday December 8

- 8:30 - 9:00 Hardy Grant, HC-1700
- 9:00 - 9:30 Alexander Jones, HC-1700
- 9:30 - 10:00 Nathan Sidoli, HC-1700
- 10:00 - 10:30 Glen Van Brummelen, HC-1700

TOM ARCHIBALD, Acadia University, Wolfville, Nova Scotia B4P 2R6  
*Jules Tannery, Paul Appell, and the mathematical research community in France, 1870-1914*

Jules Tannery (1848-1910) and Paul Appell (1855-1930) occupied key positions in the development of mathematics in late nineteenth and early twentieth century France. Tannery's early research on linear differential equations stands at the origin of much research in this area in France, undertaken by such authors as G. Floquet, E. Picard, and E. Goursat. His textbook writings, particularly the "Eléments de la théorie des fonctions elliptiques", jointly written with J. Molk, were fundamental to the French university curriculum for many years. Most important, though, was Tannery's work as the assistant director in charge of science at the Ecole normale supérieure, a post he took up in 1884. In this capacity, he exerted a strong influence on the careers of many *normaliens*. Appell, a student at the ENS shortly after Tannery, became a professor at the Faculty of Sciences in Paris and subsequently its Dean, from 1903 to 1920. Like Tannery, he was a highly influential thesis director and textbook author. As an example we may mention his "Théorie des fonction algébriques et de leurs intégrales", written jointly with Goursat and later revised by Fatou.

In this talk, I shall survey aspects of the careers of these two important figures and discuss their influence.

LAWRENCE D'ANTONIO, Ramapo College, Mahwah, New Jersey 07430, USA  
*The solution of the Beha-Eddin problem: elliptic curves to the rescue!*

Fermat's Last Theorem was not the only Diophantine problem whose solution required the use of elliptic curves. The Islamic mathematician Behā Eddīn al-'Amulī, in his early 17th century work, the *Khulāsah at al-Hisāb*, (Essence of Reckoning), discussed seven unsolved problems in algebra. One of the problems was, in fact, Fermat's Last Theorem in the cubic case. Another problem, the seventh, was to find rational solutions for the pair of equations:

$$x^2+x+2=y^2, \quad x^2-x-2=z^2.$$

In this paper we trace the history of the problem, first considering the origins of this problem and its relationship to the questions about congruent numbers. Next, the first systematic attempt at a solution, using the chord and tangent method, was done by Lucas in 1877. Finally, Horst Zimmer found the complete solution to the problem in 1983. Zimmer transforms rational solutions of the Behā Eddīn problem into rational points on a certain elliptic curve, and then gives a complete description of the group of rational points on that curve.

HARDY GRANT, York University, Toronto, Ontario M3J 1P3  
*Greek mathematics in cultural context*

I shall attempt an overview (necessarily sketchy!) of the "career" of Greek mathematics in its cultural setting. Features of the background include Eleatic philosophy, the development of the "liberal arts" tradition, the rivalry of professions, the rise of rhetoric as a technique of demonstration, and Aristotle's formalization of logic. A unifying thread is the emergence of the familiar view-not, of course, uncontroverted-of mathematics as attaining a unique exactness, certainty and insight into the nature of things.

ALEXANDER JONES, Classics, University of Toronto, 97 St. George Street, Toronto, Ontario

*Ptolemy's mathematical tones*

The Alexandrian scientist Claudius Ptolemy is best known for his celestial mechanics and his mapping of the known world. Among his less familiar books is the "Harmonics", an attempt to apply mathematical models to the esthetics of musical tones and tunings. We will describe how Ptolemy reasoned that certain kinds of ratios between whole numbers could account for the practices of the musicians of his time.

ISRAEL KLEINER, York University, Department of Mathematics and Statistics, Toronto, Ontario M3J 1P3

*Fermat: The founder of modern number theory*

I will discuss some of Fermat's major contributions to number theory, noting his intellectual debts and his legacy.

GREGORY MOORE, McMaster University, Hamilton, Ontario

*Attacking the infinite, defending the infinite: Cantor's strategy and tactics*

Georg Cantor, the creator/discoverer of set theory, was well aware of his predecessors' attitudes toward the actual infinite. Beginning in 1883, he undertook a spirited defense of the actual infinite against (as Zermelo later put it when editing Cantor's collected works) "philosophical and theological objections". We examine several of the alleged "proofs" that there is no actual infinite (including a "proof" by Cauchy), and Cantor's replies to them.

RAM MURTY, Queen's University, Kingston, Ontario K7L 3N6

*History of the Euclidean algorithm*

We will begin with Euclid and then to Aryabhata and finally to Gauss to see how the modern formulation of the Euclidean algorithm evolved over the centuries. We will also study the abstraction of the notion and address Samuel's question of the classification of rings with Euclidean algorithm.

NATHAN SIDOLI, University of Toronto

*Menelaus' Theorem in Ptolemy & Theon*

It is generally assumed that Ptolemy took the theorem upon which he built his spherical trigonometry from Menelaus' Spherics. Neither Ptolemy, nor his commentator Theon, however, makes any mention of Menelaus in this regard. We have lost most of the Greek original of Menelaus' text, in particular we have lost the section relevant to Ptolemy's spherical astronomy. Our text of Menelaus' Spherics is preserved in the Arabic tradition in a number of different translations and editions. In this paper, I examine all of the relevant versions of the Menelaus' theorem and show that the line of transmission cannot have been as straightforward as has previously been thought. In particular, I show that the Arabic tradition contains material that was not available to Ptolemy and Theon as well as material that is most likely based on their work. This technical discussion gives rise to a more speculative inquiry into the source of Ptolemy's spherical trigonometry based on the role of the Menelaus Theorem in the Spherics and the Almagest. Finally, I reconsider the possibility that Hipparchus' treatise on simultaneous risings may have used metrical methods based on the so-called Menelaus Theorem.

GLEN VAN BRUMMELEN, Bennington College

*Taking latitude with Ptolemy: Al-Kashi's final solution to the determination of the positions of the planets*

Although the model to determine planetary longitudes in Ptolemy's Almagest produced elegant and satisfactory longitude computations, his model for latitudes was, seemingly, too complicated to allow for easy handling mathematically. As a result Ptolemy was forced into making several approximations, leading to an unsatisfactory mathematical theory of latitudes. While several innovations were proposed to deal with the computation of latitudes in medieval Islam, hardly any of them dealt with the core mathematical issues. Jamshid al-Kashi, perhaps the greatest computational astronomer in the Ptolemaic tradition, achieved a complete solution to the problem in his Khaqani Zij in the early 15th century. We shall survey various Muslim contributions and describe al-Kashi's solution in detail.

THOMAS ARCHIBALD, Acadia University and Dibner Institute, MIT  
*The Reception of Fredholm's work on Integral Equations*

In 1900 Ivar Fredholm presented a method for the solution of integral equations. The method was extremely useful in showing the existence of solutions for certain types of boundary-value problems in partial differential equations, and attracted a good deal of international attention. There were two main lines of reception of the work. The more conservative consisted of detailed investigations of its meaning in the field of differential equations, with important contributions by Poincaré, Picard, T. Boggio and G. Lauricella, among others. The more radical was initiated by Hilbert, and lies at the origin of the theory of linear operators on what we now term Hilbert spaces. In this paper we examine these two threads in the early reception of Fredholm's work, which persisted as separate though interacting traditions.

RYAN BEATON, McGill  
JON BORWEIN, Dalhousie Faculty of Computer Science, 6050 University Drive  
*Philosophical Implications of Experimental Mathematics*

I will discuss the philosophical implications of my work in experimental mathematics. Philosophers have frequently distinguished mathematics from the physical sciences. While the sciences were constrained to fit themselves via experimentation to the `real' world, mathematicians were allowed more or less free reign within the abstract world of the mind. This picture has served mathematicians well for the past few millennia but the computer has begun to change this. The computer has given us the ability to look at new and unimaginably vast worlds. It has created mathematical worlds that would have remained inaccessible to the unaided human mind, but this access has come at a price. Many of these worlds, at present and perhaps for ever, can only be known experimentally. Thus, work in experimental mathematics challenges the standard view of mathematics as a subject in which proof is the sole pathway to knowledge.

David Bailey and I make this case at length in our newly published books on Experimental Mathematics ([www.expmath.info](http://www.expmath.info)). We start by observing that: One of the greatest ironies of the information technology revolution is that while the computer was conceived and born in the field of pure mathematics, through the genius of giants such as John von Neumann and Alan Turing, until recently this marvelous technology had only a minor impact within the field that gave it birth.

The future of mathematical computing is expected to rely on a mixture of symbolic and numeric (hybrid) computation that will increasingly call for significant computing power. This is obvious at the level of "grand challenge problems" in fluid dynamics, meteorology and elsewhere, but is equally true in the finance and banking sectors, as for e-commerce and, growingly, for pure mathematics. Indeed, it is my personal conviction that the success of the

computer as an inference assistant and insight generating engine demands massively parallel computation. Only when many small things are done by the computer on a real time scale (as the mathematical equivalent of a "spell-checker") can insight follow easily and freely.

LOUIS CHARBONNEAU, UQAM, C.P. 8888, Succ. Centre-ville, Montréal, QC, H3C 3P8

*L'algèbre de François Viète et sa réception en France au XVIIe siècle*

En donnant à l'algèbre son indépendance face à l'arithmétique et à la géométrie et en la situant dans le cadre d'un programme analytique, au sens des Anciens, Viète ouvre de nouvelles perspectives en mathématiques. Mais l'oeuvre du pionnier est reçue de diverses façons par les mathématiciens qui lui succèdent. Nous verrons comment progressivement la pensée viétienne est appliquée dans les travaux de ses premiers disciples, Anderson et Ghetaudi, de reconstructions d'oeuvres grecques perdues, mais aussi de disciples plus tardifs, aujourd'hui oubliés, comme James Hume, Pierre Herigone et Nicolas Durret. Leurs oeuvres illustrent le passage graduel de l'algèbre dite vulgaire à une algèbre qui nous est plus familière, comme celle de Descartes et des autres grands mathématiciens de la fin du siècle.

DAVID COYLE, Université de Montréal and Dawson College

*Henri Cartan's filters in the Bourbaki archives*

The archives for the Bourbaki group have been recently enlarged and expanded. They may be broadly divided into two categories:

- (1) Successive drafts for different sections of the final publications.
- (2) Informal communications in the form of personal correspondence and internal newsletters.

After a short description of the early Bourbaki group to provide a context for what follows, I present sample documents from the archives, focusing on integration theory and, in particular, the filters of Henri Cartan. While the more formal drafts are of central importance for the historian in tracing the development of the Bourbaki and their mathematics, it is the entertaining informal communications which provide the clearest clues to their motivations, which illustrate most clearly what it means to do mathematics, and which are in consequence of greatest interest to the student. It is, moreover, the informal items which explain the Bourbaki style of presentation, and expose most starkly the many internal contradictions of their project.

JOSEPH DAUBEN, Lehman College, City University of New York

*Suan Shu Shu (A Book on Numbers and Computation): Problems in Collating, Interpreting and Translating the Most Ancient Yet-Known Chinese Mathematical Text*

In December and January of 1983-1984, archaeologists excavating the tomb of an ancient Chinese nobleman at a Western Han Dynasty site near Zhangjiashan, in Jiangling county, Hubei Province, discovered a number of books on bamboo strips, including works on legal statutes, military practice, and medicine. Among these was a previously unknown mathematical work on some 200 bamboo strips, the *Suan Shu Shu*, or *Book of Numbers and Computation*. As the earliest yet discovered work devoted specifically to mathematics from ancient China (no later than 186 BCE), it has stirred considerable interest among historians of Chinese mathematics.

While some sections of this work are straightforward and have been understood with little disagreement, others-whether because of missing, misplaced, or incomplete parts of the texts on the bamboo strips comprising this earliest of the yet-known mathematical works from ancient China-have been open to diverse and often divergent interpretations. In some cases, related methods serve as clues to help interpret the meaning of given problems, as is the case for the three problems devoted to *Fu Tan*, *Lu Tang*, and *Yu Shi*. But for another pair of seemingly related problems, *Yi Yuan Cai Fang* and *Yi Fang Cai Yuan*, there has been little agreement about whether these are inverse or quite different problems, and virtually everyone who has approached these two problems has understood them differently in trying to account for the statements, answers, and methods given for these two problems. This presentation will be devoted to discussion of the various collations and explanations offered for these especially challenging parts of the *Suan Shu Shu*, and what they may tell us about early Chinese mathematics in general.

ROBERT DAWSON, St Mary's University, Halifax, NS B3H 3C3

*The Slide Rule in the 21st Century Classroom: Cultural Icon and Manipulative*

When many of the parents of today's high school students were themselves in high school, the slide rule was ubiquitous: mysterious to many, but as iconic a badge of the scientist as the doctor's stethoscope or the farmer's pitchfork. Today, three decades later, not a single one of the famous brands-Keufel and Esser, Hemmi, Faber and Castell, *etc.*-are still manufactured. For the inflation-adjusted cost of a basic slide rule in 1970, you can buy a calculator which can do everything that the slide rule could, and much more. Moreover, the calculator is easy enough to use that it has achieved a level of penetration into the schoolroom that the slide rule never did.

Nonetheless, it can be argued that the slide rule had pedagogical potentials that the calculator-no matter how sophisticated-does not. It makes many of the properties of exponents and logarithms clear in an intuitive, tactile way. It encourages order-of-magnitude calculation and provides a good motivation for scientific notation. And, finally, it is today a link with living mathematical history; for some decades yet, many school classes will be able to find a parent or grandparent who used a slide rule professionally.

This talk suggests some ways in which the slide rule might be included in the modern high school classroom. In particular, I will look briefly at ways of getting around the current (and, it is to be feared, continuing) slide rule shortage, and some ways in which the design of the slide rule might be modified for an environment in which it is primarily a manipulative rather than a calculating device.

PAUL DEGUIRE, Université de Moncton  
*Teaching history of maths to non scientists*

For the past 6 years I have presented conferences to groups of liberal art students whose formation includes no maths and no science beyond high school. They are students in a specific 1 year program called "Odyssée Humaine" in which all the different classes are interrelated, that is the teacher in a philosophy class for instance knows exactly where the students are in their geography or history class and builds his own course upon this knowledge.

We are in the process of creating a specific course on History of Maths and Science that will be integrated into a modified version of l'Odyssée Humaine that is supposed to begin in September 2005. Of course, it will probably take a few years to create a satisfactory description of this course. Nevertheless, it starts next September and in this talk, I'll present the starting point of my thoughts concerning this presentation of Math History to non-scientists.

One of the important difference between this course and a similar course that is intended for math students is the objective. Knowledge of specific results is not an important part of this new course, but the integration of mathematical knowledge and scientific knowledge in the broader scope of the evolution of the western culture is.

Two examples: First, Mathematics in Ancient Greece probably played an important rôle in the apparition of Greek rationalism and philosophy. Second, new successes in maths, applied as well as pure, played an important part of the fast evolution of Europe after the Middle Ages. Just knowing that they were able to do things that the Ancient Greeks and Romans never did was enough to create a climate of confidence that lead to the scientific revolution of the 17th century.

HAROLD HASTINGS, MARYSIA WEISS, YIHREN WU, Department of Physics,  
151 Hofstra University, Hempstead, NY 11549-1510  
*A Century of Topological Dynamics-reflections on its impact upon education and research*

Many of us were drawn to topological dynamics in the 1970s and 1980s by the apparently surprising behavior and broad applicability of maps of the unit interval: a sequence of bifurcations leading to the onset of chaos showing apparently



random behavior. At the same time, many key results are readily accessible to undergraduate students, and topological dynamics has found its way into the undergraduate curriculum. This talk will survey some high points of historical development of topological dynamics.

Partially supported by the NSF.

MICHEL HELFGOTT, East Tennessee State University, Department of Mathematics,  
P. O. Box 70663, Johnson City, TN 37614, USA  
*Thomas Simpson and Problems of Maxima and Minima*

Thomas Simpson (1710-1761) was a self-taught English mathematician who wrote several remarkable books, among them "The Doctrine and Application of Fluxions" (1750). The fact that in 1823 it was still being published attests to its wide and deserved popularity. Changing the word "fluxion" by "derivative", the problems on maxima and minima that can be found in the above-mentioned work stand out as particularly interesting and can be read with profit nowadays. Keeping a historical framework we will discuss some of these problems.

ROBERT IRWIN, SUNY Oswego, Oswego, NY 13126  
*Early Computability Theory*

A survey of early computability theory will be presented. Here, "early" means the 1920s and 1930s. The main different, but equivalent, approaches to formalizing (or defining) the notion of effective calculability in this period will be discussed, as will be the motivations of the prime movers of the theory, including Gödel, Kleene, Church, Turing and Post. Modern continuations of some research threads originating in the early period will be briefly discussed.

ALEXANDER JONES, Classics, University of Toronto, 97 St. George Street  
*Patterns of deduction in Ptolemy's Almagest*

Ptolemy's treatise on celestial mechanics, the Almagest, is not composed as an assemblage of reasonings but as a single reasoning, the biggest deductive argument in all ancient applied mathematics. This is exceptional in the history of astronomical writing, even (so far as we can tell) in antiquity. Other writers recognize that the solutions of some problems depend on others, but they generally assume that some degree of isolation between problems is possible. But for Ptolemy, mathematical astronomy is like a puzzle that can only be taken apart piece by piece in a particular order starting with a particular key piece.

This paper will consider three aspects of the logical structure of the Almagest:

- (1) the three levels of argument (nontechnical deduction, technical quantitative deduction, and model confirmation);

- (2) the large-scale ordering of the topics; and
- (3) the use of recursive deductions.

NIKY KAMRAN, McGill University, Montreal, Quebec, Canada  
*Einstein, Hilbert and the field equations of gravitation*

It is a remarkable fact that Einstein and Hilbert discovered the field equations of the relativistic theory of gravitation almost simultaneously, following totally different paths. I shall review some of the highlights of this fascinating episode, including the important contribution that Emmy Noether made to the development of Hilbert's approach.

JIM LAMBEK, Mathematics Department, McGill University  
*Remarks on the History of Categorical Grammar*

Categorical grammar is an attempt to place most of the grammar of a natural language into the dictionary. This is accomplished by assigning to each word in the dictionary one or more types, which are taken to be elements of an algebraic system or terms of a substructural logic. There are, at present, two streams of investigation. One, going back to Ajdukiewicz, relies on residuated monoids, *i.e.*, monoids equipped with two binary operations of division that mimic logical implication. The other makes use of two unary operations instead, which mimic the logical operation of negation. It turns out that the second approach, which I now favour, has historical roots in ideas of C. S. Peirce and Z. Harris.

DUNCAN MELLVILLE, St. Lawrence University  
*Multiplication in Mesopotamia*

We trace the development of the ideas of multiplication in Mesopotamia from the first archaic evidence to the Old Babylonian period (2000-1600 BC). In particular, we will show how the abstract sexagesimal place value system that was developed at the end of the third millennium allowed the unification of ideas of area and repeated addition by freeing these concepts from the metrological systems that had earlier been in use.

#### ROUND TABLE DISCUSSION

CHRISTIANE ROUSSEAU, Université de Montréal  
*From Rolle's theorem to Khovanskii theory of fewnomials*

The modelling of an important number of problems in mathematics leads to real systems of equations with real unknowns. Depending on the context we may only be interested in the number of solutions, or the subproblems of, either finding a bound for the number of solutions, or showing that the number of solutions are finite. In this lecture I will discuss the power of Rolle's theorem and its generalization by Khovanskii to the Rolle's theorem for dynamical systems. I will introduce the theory of fewnomials and discuss some applications.

NATHAN SIDOLI, University of Toronto  
*Geometrical Analyses in Heron's Measurements*

The bulk of Heron's *Measurements* can be described as applied, perhaps even practical, mathematics. In the third book, however, he gives a series of analytic propositions in the pure geometric idiom. He tells us that these will be useful for solving problems which cannot be solved by computation (*dia ton arithmon*). An investigation of this material is interesting on two counts. In the first place, it adds to our limited evidence for the actual practice of ancient geometrical analysis. More interestingly, however, it makes clear that Heron considered geometrical analysis capable of solving certain problems that were not susceptible to computation. My talk will try to elucidate Heron's thinking in these matters.

GLEN VAN BRUMMELEN, Bennington College  
*Al-Samaw'al's Curious Approach to Trigonometry*

The 12th-century mathematician Ibn Yahya al-Maghribi al-Samaw'al, now better known for his algebra, also wrote the extensive treatise *Exposure of the Errors of the Astronomers*. This fascinating under-studied work, containing criticisms of a number of astronomers, provides an interesting study of debates over the proper practice of medieval astronomy. In particular, al-Samaw'al eschews any form of geometrical approximation, no matter how trivial. One of his objections is to the methods that had been used to determine the geometrically unattainable sin (1 degree), in Ptolemy's *Almagest* as well as in later Muslim works. To avoid this apparently unavoidable problem, al-Samaw'al presents an alternate trigonometric table that breaks the circle into 480 rather than 360 parts. We shall present the table as well as one of its uses in al-Samaw'al's work.

ALEXEI VOLKOV, Université du Québec à Montréal  
*History of traditional Vietnamese mathematics: the state of the field*

The history of traditional Vietnamese mathematics can be reconstructed on the basis of the extant mathematical treatises only partially. Some 20 mathematical treatises are to be found in Vietnamese libraries, of which the latest ones were compiled in the early 20th century; as for the earliest texts, the date of their actual compilation remains uncertain, but most likely they do not antedate the early 18th century. All these texts are compiled on the basis of Chinese mathematical treatises of the Ming (1368-1644) and Qing (1644-1911) dynasties.

The paper will focus on several issues pertaining to the study:

- (1) the legends of the life and scientific activities of the state functionary Luong The Vinh (1441-1496?) conventionally viewed as the most outstanding Vietnamese mathematician;
- (2) the scientific interaction between Vietnamese literati and Jesuits in the

early 17th century;  
(3) the revival of mathematics education in Vietnam in the early 18th and 19th century; and  
(4) the decline of traditional mathematics in the context of introduction of European-style mathematics education under French colonial rule.