CSHPM/SCHPM
Annual Meeting/Colloque Annuel
Université du Québec à Montréal (UQAM)
Montréal, Québec

Monday (2018-06-04):

9:30-9:45 Welcome by Dirk Schlimm (President of CSHPM/SCHPM)(Room: PK-R220)[1]

Session 1: Medieval Philosophy & Astronomy
Room: PK-R220; Presiding: Maria Zack

9:45-10:15 Glen Van Brummelen (Quest University), "Jamshīd al-Kāshī's Tables of Planetary Latitudes"

Coffee Break

Session 2: Mathematics in Russia & Ukraine
Room: PK-R220; Presiding: Craig Fraser

10:30-11:00 Mariya Boyko (University of Toronto), "The Reception and Criticism of the Soviet Mathematics Curriculum Reform in the Late 1970s: Factors that Led to Curriculum Counter-Reforms"

11:00-11:30 Inna Tokar (City College of New York), "History of Mathematics Education for Gifted Students in the Former Soviet Union"

11:30-12:00 Maryam Vulis (City University of New York), "Ukrainian Mathematicians of the 19th-20th Centuries and their Contributions to the Development of Mathematics and Mathematical Culture in Ukraine"

Lunch Break

12:00-14:00 Executive Council Meeting (Room: TBA)

[1] Note that all talks are on the ground floor (the "R" in the code) of *Pavillon Président Kennedy* (code: PK) at the corner of Avenue du Président Kennedy and Rue Jeanne-Mance.

Session 3: Proof and Practice
Room: PK-R220; Presiding: Jean-Pierre Marquis

14:00-14:30 Bernd Buldt (Indiana University—Purdue University Fort Wayne), "Mathematical Practice and Phenomenology"

14:30-15:00 Robert Hudson (University of Saskatchewan), "Conditional Independence and the Value of Diverse Evidence"

15:00-15:30 Zoe Ashton (Simon Fraser University), "Audience-Reflective Proof: A Case Study from Knot Theory"

15:30:16:00 Robert Thomas (University of Manitoba), "Why Mathematical Style Does not Need Philosophical Justification"

Tuesday (2018-06-05):

Session 4a: History of Logics (Parallel Session)
Room: PK-R210; Presiding: Greg Lavers

8:45-9:15 Dirk Schlimm (McGill University), "What was Boole's System of Logic about?"

9:15-9:45 V. Frederick Rickey (United States Military Academy), "Professor Bolesław Sobociński and Logic at Note Dame"

9:45-10:15 Daniel Lovsted (McGill University), "Logical Incommensurability in the 20th Century: Fred Sommers and his Contemporaries"

Session 4b: History of Seventeenth-Century Mathematics (Parallel Session)
Room: PK-R605; Presiding: Rob Bradley

8:45-9:15 George Heine (Independent Scholar) "Two Brothers and the Lemniscate"

9:15-9:45 Christopher Baltus (SUNY College at Oswego), "Philippe de la Hire: Was he Desargues' *Schüler*?"

9:45-10:15 Maria Zack (Point Loma Nazarene University), "Everyone's Favorite Curve: The Cycloid"

Coffee Break

Session 5a: History of Logics
Room: PK-R210; Presiding: Elaine Landry

10:30-11:00 Jean-Pierre Marquis (Université du Montréal à Québec), "Bourbaki's Structuralism: Its Evolution and Impact"

11:00-11:30 Greg Lavers (Concordia University), "Carnap, Turing and the Paradox of Analysis"

11:30-12:00 David Orenstein (University of Toronto), "Complementary Duality: The Bertrand Russell Archives at McMaster University and the Slater/Walsh Philosophy Collection at the University of Toronto"

Session 5b: History of Eighteenth-Century Mathematics
Room: PK-R605; Presiding: Amy Ackerberg-Hastings

10:30-11:00 Robert Bradley (Adelphi University), "Did D'Alembert Really Believe in Limits?"

11:00-11:30 Cameron Friend (Quest University) "Carrying Across: A Translation and Analysis of Leonhard Euler's Text "*Problematis Cuiusdam Pappi Alexandrini Constructio*," and the Accompanying Text by Nicolaus Fuss"

11:30-12:00 Craig Fraser (University of Toronto), "Euler and Divergent Series: Some Historiographical Reflections"

Lunch Break

12:00-14:00 Annual General Meeting (Room: PK-R605)

14:00-15:00 Annual CSHPM Kenneth O. May Lecture: Emily Grosholz (Pennsylvania State University), *How Number Theory and Logic Benefit from Productive Ambiguity: Gödel, Mazur, Wiles and Macintyre* (Room: PK-R605)

Session 6: Special Session—History of Philosophy of Mathematics
Room: PK-R605; Presiding: Dirk Schlimm

15:15-15:45 Yousuf Hasan (University of Western Ontario), "Applying Carnap's Internal/External Distinction to Mathematics"

15:45-16:15 James T. Smith (San Francisco State University) (joint work with Elena A. Marchisotto (California State University—Northridge)), "Intermezzo"

16:15-16:45 Osama Eshera (McGill University), "Theoretical Mathematics in Avicenna's Metaphysics"

Wednesday (2018-06-06):

Session 7: History of Mathematics in the Classroom
Room: PK-R220; Presiding: George Heine

9:15-9:45 Janet Heine Barnett (Colorado State University—Pueblo), "A Gaussian Tale for the Classroom: Lemniscates, Arithmetic-Geometric Means, and More"

9:45-10:15 Jonathan Seldin (University of Lethbridge)(joint work with Fairouz Kamareddine (Heriot-Watt University)), "Using the History of Mathematics to Teach the Foundations of Mathematical Analysis"

Coffee Break

Session 8a: History of Nineteenth-Century Mathematics
Room: PK-R220; Presiding: Eisso Atzema

10:30-11:00 Amy Ackerberg-Hastings (Independent Scholar), "Charles Davies as a Philosopher of Mathematics Education"

11:00-11:30 Maritza Branker (Niagara University), "Cauchy's Persuasive Appeal"

11:30-12:00 Roger Godard (Royal Military College of Canada) (joint work with John De Boer (Royal Military College of Canada)), "Gauss and the Forgotten Model of the Earth's Magnetic Field"

Session 8b: On the Axiom of Choice
Room: PK-R210; Presiding: Robert Thomas

10:30-11:00 Valérie L. Therrien (University of Western Ontario), "The Axiom of Choice and the Road Paved by Sierpiński"

11:00-11:30 Aaron Thomas-Bolduc (University of Calgary) & Eamon Darnell (University of Toronto), "Strengthening Truth"

11:30-12:00 Elaine Landry (University of California—Davis), "Mathematics is not Metaphysics"

LUNCH BREAK

Session 9a: Philosophie des Mathématiques en Français
Room: PK-R220; Presiding: Jean-Pierre Marquis

14:00-14:30 Aurélien Jarry (Bergische Universität Wuppertal), "L'Équivalence entre Catégories: A Third Way of Analogy?"

14:30-15:00 Jean-Charles Pelland (Université du Québec à Montréal), "Arithmetic, Culture, and Attention"

Session 9b: On Applied Mathematics
Room: PK-R210; Presiding: Craig Fraser

14:00-14:30 José Perez Escobar (ETH Zürich), "Mathematical Modeling in Physics, Biology and Their Intersection: Differences in the Use of Mathematical Tools across the Empirical Sciences

14:30-15:00 Toby Reid (University of Toronto), "Early Pedagogy of General Relativity Theory in the USA for American Relativistic Cosmology in the 1920s and 1930s"

15:30-15:45 CONCLUDING REMARKS (Room: PK-R220)

End 2018 CSHPM/SCHPM Annual Meeting

ABSTRACTS

Amy Ackerberg-Hastings, Independent Scholar (aackerbe@verizon.net), **Charles Davies as a Philosopher of Mathematics Education**

Charles Davies (1798-1876), who taught at West Point, Hartford's Trinity College, New York University, and Columbia, was one of the most prolific and popular compilers of mathematics textbooks in the United States in the 19th century. This talk explores his 1850 *The Logic and Utility of Mathematics, With the Best Methods of Instruction Explained and Illustrated*, which James K. Bidwell and Robert G. Clason (1970) and Phillip S. Jones and Arthur F. Coxford, Jr., (1970) called the "first American book on mathematics teaching methods." In addition to providing an overview of the contents of this volume and placing it within the context of mid-19th-century mathematics education, I will consider Davies's general conception of mathematical knowledge as well as the extent to which that conception was original.

Zoe Ashton, Simon Fraser University (zashton@sfu.ca), **Audience-Reflective Proof: A Case Study from Knot Theory**

The role of audiences in mathematical proof has largely been neglected, in part due to misconceptions originating in Perelman & Olbrechts-Tyteca (1969) which bar mathematical proofs from bearing reflections of audience consideration. I argue that mathematical proof is typically argumentation and that a mathematician develops a proof with his universal audience in mind. In so doing, he creates a proof which reflects the standards of reasonableness embodied in his universal audience. Given this framework, we can reconstruct the introduction of proof methods based on the mathematicians likely universal audience. In this paper, I primarily focus on using that framework to examine a case study from Alexander and Briggs's work on knot invariants. Alexander and Briggs (1927) use a dotting notation to indicate crossings in knot diagrams. The dotting notation is a change from the more common "break" notation and it is relied on throughout their proofs. I argue that Alexander's dotting notation is a proof method that reflects the mathematician's universal audience.

Christopher Baltus, SUNY College at Oswego (christopher.baltus@oswego.edu), **Philippe de la Hire: Was he Desargues' *Schüler*?**

Philippe de la Hire (1640—1718) was the third 17th century pioneer of projective geometry, after Girard Desargues (1591—1661) and Blaise Pascal (1623—1662). Desargues' groundbreaking work of 1639, on conic sections, notoriously difficult and confusing, issued in just 50 copies, had disappeared within 20 years. But then La Hire, son of a friend of a friend of Desargues, issued his own projective treatment of conic sections in 1673. Could it really be independent? To add to the intrigue, only to be learned in 1845, La Hire made a transcript, in 1679, of Desargues' booklet, claiming to have only seen that work in 1679. Really? The word "Schüler" is from Lehmann's 1888 *De la Hire und seine Sectiones conicae*. More recent accusations are from Rene Taton. Except that in examining the work itself, as in this presentation, La Hire's claim to independence is plausible. We'll look at the treatment of the pole/polar relation.

Janet Heine Barnett, Colorado State University—Pueblo (janet.barnett@csupueblo.edu), **A Gaussian Tale for the Classroom: Lemniscates, Arithmetic-Geometric Means, and More**

In a July 1798 entry in his mathematical diary, Gauss wrote: "On the lemniscate, we have found out the most elegant things exceeding all expectations and that by methods which open up to us a whole new field ahead." Paving the way to the new field of elliptic integrals predicted by Gauss was an elegant relationship that he discovered between three particular numerical values: π, an integral value associated with the arc length of a lemniscate $\left(\int_0^1 \frac{1}{\sqrt{1-t^4}}dt\right)$, and the arithmetic-geometric mean of 1 and $\sqrt{2}$. As an example of the powerful role which analogy and numerical experimentation can play within mathematics, the tale of Gauss' path to these discoveries is one well worth sharing with today's students. This talk describes a series of brief "Primary Source Projects" based on excerpts from Gauss' mathematical diary and other related manuscripts designed to tell that tale, while also serving to consolidate student proficiency with several standard topics from the traditional Calculus II course.

Mariya Boyko, University of Toronto (mariya.boyko12@gmail.com), **The Reception and Criticism of the Soviet Mathematics Curriculum Reform in the Late 1970s: Factors that Led to Curriculum Counter-Reforms**

In 1958 the Soviet government led by Nikita Khrushchev initiated a major reform of education in order to bridge the gap that then existed between the school curriculum and the practical needs of the state. Prominent mathematicians and educators - led by Andrei Kolmogorov - were involved in re-writing the mathematics curriculum. However, the content of the new curriculum proved to be unsuitable for the general audience of students who were not highly interested in pure mathematics. By the late 1970s, students who were the product of Kolmogorov curriculum reform were entering post-secondary institutions. Many of them performed poorly in technical and computational tasks at the entrance exams. This fact was of substantial concern for school teachers and professional mathematicians alike. The resulting unrest in the community of educators and mathematicians led to the introduction of mathematics curriculum counter-reforms, led by prominent mathematician Ivan Vinogradov, and public criticism of Kolmogorov's contributions to Soviet mathematics education. However, mathematics teachers were not fully satisfied with the counter-reformed curriculum either. In this talk, I will show that the teachers' dissatisfaction with the changes in the mathematics curriculum proposed by the Academy of Pedagogical Science was not based so much on the content of the curriculum, as it was on the overall setup and timeline of the changes in mathematics education. The teachers were overwhelmed with new textbooks that were coming out every year, and perceived the counter-reform as another unnecessary experiment, rather than help. We will examine excerpts from teachers' comments and criticisms published in a prominent journal Mathematics in the School to expose the underlying reasons for teachers' dissatisfaction with both the Kolmogorov reforms and the Vinogradov counter-reforms.

Robert Bradley, Adelphi University (bradley@adelphi.edu), **Did D'Alembert Really Believe in Limits?**

Jean d'Alembert (1717—1783) is generally considered to be an important participant in the evolution of the differential and integral calculus. His role as an early champion of the limit concept was influential in its gradual adoption for the foundations of calculus. However, none of his books or research papers make any mention of limits—his only writings on the subject were the articles on "differential" and "limit" in Diderot's Encyclopédie. In this talk we will endeavor to determine in what sense d'Alembert believed that the theory of limits was "the basis of the true metaphysics of the differential calculus."

Maritza Branker, Niagara University (mbranker@niagara.edu), **Cauchy's Persuasive Appeal**

Augustin Louis Cauchy has been described as the true founder of complex analysis. His 1821 textbook *Cours d'analyse* was extremely influential in the development of the field and the excellent translation by Robert E. Bradley and C. Edward Sandifer allows us to appreciate its similarities to a modern introductory analysis text in English. This talk will discuss the approach taken by Cauchy and the impact on his readers in his time and ours.?

Bernd Buldt, Indiana University—Purdue University Fort Wayne (buldtb@ipfw.edu), **Mathematical Practice and Phenomenology**

Husserl's philosophy of mathematics has recently become the focus of much scholarly interest among philosophers. My own approach, however, rather follows the lead of Gian-Carlo Rota and reflects two assumptions. First, Husserl's thinking about mathematics and its objects was informed by his own experience as a mathematician and the discussions in the mathematical community of the time (i.e., Hilbert's Göttingen and beyond). Second, the mathematical experience has changed since Husserl's time and that therefore current ideas about mathematical practice must inform both analysis and discussion. Thus, cognizant of recent scholarship on Gödel and Husserl but guided by a different approach I investigate the extent to which the language of phenomenology, in particular of the later Husserl, provides a coherent and adequate framework for understanding, first, the individual mathematical experience (i.e., learning, applying, and proving mathematics) and, second, the mathematical practice as embodied in a community of peers. Given the limited time, the main focus will be on mathematical concepts.

Osama Eshera, McGill University (osama.eshera@mail.mcgill.ca), **Theoretical Mathematics in Avicenna's Metaphysics**

Very little scholarly attention has been devoted to the philosophy of theoretical mathematics in the history of Islamic science. This is especially true of number theory and theoretical geometry where the most significant scholarship to date, Hassan Tahiri's *Mathematics and the Mind*, focuses on the epistemic place of theoretical mathematics in Avicenna's (d. 1037) philosophy but neglects more foundational questions on the metaphysics and ontology of mathematical objects. I aim to address this scholarly gap by posing three questions in the context of the metaphysics of al-Fārābī (d. 951) and Avicenna: What is the nature of number? What kind of existence do numbers have? What is the metaphysical status of geometrical objects? Through this line of questioning, I examine the complex metaphysics of the algebraic 'thing' (*shay'*), a single object that can potentially be different species (e.g., number, magnitude, or shape). I intend to show that the development of Avicenna's metaphysical system, against al- Fārābī's in the background, is shaped, at least in part, by the demands of algebra as an nascent mathematical discipline. Ultimately, this paper is a case study in the reciprocal relationship between mathematics and philosophy in the history of Islamic thought.

Craig Fraser, University of Toronto (craig.fraser@utoronto.ca), **Euler and Divergent Series: Some Historiographical Reflections**

Euler's work on divergent series and its relation to the later theory of summability raise issues of historical interpretation. Some historians have called attention to modern elements in Euler's understanding of series. For example, Barbeau and Leah (1976) write "Euler frequently makes it clear that he is cognizant of the behaviour of infinite series and, in fact, distinguishes between convergent and divergent series along modern lines." However, some of the prominent architects of the modern theory of divergent series such as G. H. Hardy have emphasized striking differences between Euler's understanding and the modern one. Recent studies by such historians as Ferraro (2008) have also called attention to historically particular characteristics of the eighteenth-century understanding of series. The paper examines Euler's understanding and treatment of divergent series and explores historiographical issues involved in an evaluation of his work in this area.

Cameron Friend, Quest University (cameron.friend@questu.ca), **Carrying Across: A Translation and Analysis of Leonhard Euler's Text "*Problematis Cuiusdam Pappi Alexandrini Constructio*," and the Accompanying Text by Nicolaus Fuss**

One of the best mathematicians of all time, Leonhard Euler shaped almost every part of the mathematics of his time. We shall focus on one of his geometric works, written near the end of his life, Problematis cuiusdam Pappi Alexandrini Constructio (translated with Cynthia Huffman). This paper, solving a seemingly simple geometrical problem, has connections to four different historical periods from Euclid to Pappus, from Euler to Steiner. In particular, the paper is part of the story of an important transition in conceptions of geometry in the late 18th century to the early 19th century, originally explored by Michael Fried, wherein functional mappings between points and numbers became an important aspect of geometrical practice.

Roger Godard, Royal Military College of Canada (rgodard3@cogeco.ca) (with John De Boer, Royal Military College of Canada (john.deBoer@rmc.ca)), **Gauss and the Forgotten Model of the Earth's Magnetic Field**

In 1839, Carl Friedrich Gauss published his famous article on the modeling of the terrestrial magnetic field. Beneficiating from the previous scientific knowledge about the gravitational theory, Gauss assumed that the earth is surrounded by a magnetic potential which obeys to the Laplace equation. And Gauss solved this equation in spherical coordinates. Gauss assumed a trial solution of the type where he generated a solution as a series of associated Legendre polynomials. In order the compute the coefficients in the spherical harmonic expression for the potential, Gauss selected 189 equations for 24 unknown coefficients that he found by the method of least squares. Indeed, Gauss assumed that the magnetic potential V goes to zero as the radial radius goes to infinity. Unfortunately, the magnetic potential is unknown at the earth surface, and only the three components of the earth magnetic field are accessible. Therefore, Gauss needed data from terrestrial magnetic observatories. We shall give a brief historical survey of magnetic observations, followed by comments on the gravitational theories, mainly from the works of Laplace and Legendre and Green's potential theory. Finally we shall examine the validity of Gauss' approach and his results.

Emily Grosholz, Pennsylvania State University (erg2@psu.edu), **How Number Theory and Logic Benefit from Productive Ambiguity: Gödel, Mazur, Wiles and Macintyre**

I briefly revisit H. B. Enderton's *A Mathematical Introduction to Logic* (1972), and Nagel and Newman's account of Gödel's Proof (1958), before examining the first stage of Wiles' proof of Fermat's Last Theorem. I turn to Jeremy Butterfield's critique of the mid-twentieth century model of theory reduction first to show that the limitations of the theory reductions presented by Enderton (and Gödel) are, in Butterfield's own terms, "too weak and too strong." I argue secondly that these 'reductions,' better understood as intersections or superpositions, lend themselves via their productive ambiguity to the growth of knowledge. I give a novel reading of Gödel's incompleteness results, arguing that Gödel, to carry out his proof, had to use modes of representation that lend themselves to logical analysis (Russell's notation) but not to computing or referring, and at the same time other modes of representation that lend themselves to successful reference (Indo-Arabic/Cartesian notation). He must use disparate registers of the formal languages available to him, combine them, and exploit their ambiguity. Then in my discussion of Wiles' proof, I note that the proof has been extensively analysed by mathematical logicians, notably Angus Macintyre, and that Macintyre is interested in questions different from those that concern Wiles. Macintyre is, for instance, interested in whether the proof can be carried out within first order Peano Arithmetic. This superimposes a new, logical discourse on Wiles' discourse, which involves integers, rational numbers, modular forms and elliptic curves. The interaction of this new discourse with the old one may give rise to the growth of mathematical knowledge. Indeed, the interaction between logic and number theory may give rise to novel objects, procedures and methods still to be discovered.

Yousuf Hasan, University of Western Ontario (yhasan3@uwo.ca), **Applying Carnap's Internal/Ex-**

ternal Distinction to Mathematics

In *Empiricism, Semantics, and Ontology* (1950), Carnap distinguishes between "internal" and "external" questions. Internal questions, for Carnap, arise within a language and are amenable to the ordinary methods of proof relative to that language. In contrast, external questions should be understood as practical questions about adopting a language. While Carnap originally made this distinction to sanction talk of abstract entities as an empiricist (1950), he later extended his distinction to mitigate realist/instrumentalist disputes with respect to theoretical entities (1958, 1966/1974). It has been argued, however, that Carnap's distinction is to be abandoned since it would make the reality of atoms a practical matter of choosing an "atom language" (Maddy 2008). The case of atoms is especially important since Carnap's distinction would seem to undermine the importance of Einstein and Perrin's works that decisively settled the debate between energeticists and atomists in favour of the latter. I will answer the question, "Why is the application of Carnap's distinction less straightforward in the case of theoretical entities such as atoms than mathematical entities?" I will expand on what has recently been suggested by William Demopoulos: the entities of physics are favoured preanalytically in a way that mathematical entities are not (2011).

George Heine, Independent Scholar (gheine@mathnmaps.com), **Two Brothers and the Lemniscate**

In the last decade of the seventeenth century, Jacob and Johann Bernoulli independently discovered the lemniscate. Using original sources, we explore the contributions of each, and speculate on how their separate works may have contributed to sibling rivalry.

Robert Hudson, University of Saskatchewan (r.hudson@usask.ca), **Conditional Independence and the Value of Diverse Evidence**

It is a commonly espoused methodological principle that evidence for a hypothesis is better if evidence is drawn from a variety of independent sources. In this paper I examine a probabilistic argument for this principle advanced by Elliot Sober in his 2008 book Evidence and Evolution. A similar argument is contained in the much-cited Condorcet Jury Theorem (CJT). Each of these arguments utilizes a form of probabilistic independence called 'conditional independence.' The underlying idea behind Sober's argument and CJT is that, if probabilistically independent evidential reports converge in their testimony, this provides better evidential support for a hypothesis than if these reports are probabilistically dependent. But as I show, the use of conditional independence in this context leads to the result that similar evidential benefits are derived with the repeated use of a single evidential source. I take this to be a reductio ad absurdum of the value of such probabilistic arguments based on conditional independence. The upshot is that if we wish to demonstrate the epistemic value of diverse evidential sources we need to look elsewhere than applying conditional independence. I close by discussing some possibilities along these lines.

Aurélien Jarry, Bergische Universität Wuppertal (jarry@uni-wuppertal.de), **L'Équivalence entre Catégories: A Third Way of Analogy?**

Recouvrant en sciences cognitives un principe fondamental et omniprésent de la pensée, à l'œuvre même dans la création des concepts mathématiques, la notion d'analogie connaît ces dernières décennies un grand regain d'intérêt en histoire et philosophie des mathématiques et des sciences en général. Schlimm distingue deux modèles descriptifs de l'analogie, définie comme relation de similarité entre deux domaines. Le premier, prédominant en sciences cognitives, explique l'analogie en termes de préservation/projection de structure ("structure-mapping"), le deuxième en termes de lois ou axiomes communs. Schlimm souligne la pertinence du second en mathématiques pour décrire le processus d'abstraction, dont Marquis propose une explication plus élaborée. En m'inspirant de Brown/Porter, je défends l'introduction d'un troisième modèle, irréductible aux deux autres et nécessaire pour décrire l'identité/équivalence algèbre-géométrie formulée en géométrie algébrique contemporaine dans le langage des catégories. Cette "identité" justifie l'emploi du terme analogie pour désigner une relation de correspondance entre "structures" non-isomorphes, dont les lois ou relations ne sont pas communes mais se reflètent en miroir.

Elaine Landry, University of California—Davis (emlandry@ucdavis.edu), **Mathematics is not Metaphysics**

I will critically examine the claim, typically made against structural realists, that one cannot metaphysically speak of structural relations without individuals as structured relata, because relata are prior to relations. Specially, I will argue that such claims result from a misunderstanding of set theory, model theory and, most problematically, a general conflation of mathematics with metaphysics. For example, it has been argued by Bueno (2017) that the ZFC axiom of extensionality implies individuality, i.e., implies that set relations arise from individuals as elements, so that individuals as elements are prior to set relations. First, this claim misunderstands that ZFC is a theory about sets, i.e., all the objects in the universe of discourse are sets, so there are no "elements" that are not sets. Second, even if one were to shift to a Suppesian urelement account of sets, which allows for talk of individuals as elements of sets, the metaphysical claim of the priority of individuals is not thereby established, because the axiom of extensionality does not hold for a urelement set theory. A similar argument from model theory will be used to show that taking models as structures does not thereby imply that objects are prior to structural relations.

Greg Lavers, Concordia University (glavers@gmail.com), **Carnap, Turing and the Paradox of Analysis**

In 1942 C. H. Langford published a paper in the Schilpp volume on G. E. Moore that questions the possibility of giving a successful analysis. Langford's paper contains the first published mention of the phrase 'paradox of analysis'. Langford argued that any analysis must be either uninformative, if the analysandum and analysans have the same meaning, or incorrect otherwise. Rudolf Carnap saw this paradox as ruling out a certain view of analyses. The condition of correctness is too strong, and an explication (his term for analyses) must introduce a new notion. This notion of an explication becomes a cornerstone of Carnap's philosophy. In his 1937 paper Alan Turing gives an analysis of the notion of what is effectively computable. Turing's analysis is provably equivalent to others, including Alonzo Church's analysis which slightly predated Turing's, and has been singled out, by Gödel for example, as being a particularly successful analysis. In fact, Turing's analysis seems to be successful in exactly the way that the paradox of analysis appears to rule out. That is, it is largely seen as both correctly capturing the intuitive notion of effective computation, and at the same time informative. In this paper I will identify what it is about Turing's analysis that allows him to avoid the paradox of analysis. I will also identify lessons to be drawn from this case for a Carnapian.

Daniel Lovsted, McGill University (daniel.lovsted@gmail.com),**Logical Incommensurability in the 20th Century: Fred Sommers and his Contemporaries**

This paper is one piece in a larger project that seeks to understand the history of logic in Kuhnian terms, i.e., as consisting of periods of paradigm-guided practice interrupted by scientific revolutions. Broadly dividing the history of logic into periods of Aristotelian and Fregean practice, I consider the shift between these two periods as a true Kuhnian revolution. Crucially, a Kuhnian revolution involves incommensurability, which I interpret as the divergence of evaluative standards across a revolutionary divide, and which leads to the malfunction of debate between practitioners of different paradigms. In this talk, I propose that we can see evidence of incommensurability in the debates between American logician Fred Sommers (1923-2014) and his contemporaries. The debates between Sommers, who developed a neo-Aristotelian logical system, and his exclusively Fregean interlocutors, show trends of misunderstanding, frustration, and inability to converge on standards of judgment that indicate a level of deep incompatibility between their practices. I focus on three issues of debate—a specific syntactic issue, notation, and semantic theory—and demonstrate how the divergences between Sommers and his contemporaries are embedded in their paradigms in fundamental ways. In this way, I hope to advance a Kuhnian history of logic and contextualize Sommers' individual career.

Jean-Pierre Marquis, Université du Montréal à Québec (jean-pierre.marquis@umontreal.ca), **Bourbaki's Structuralism: Its Evolution and Impact**

In this talk, I will look at the evolution of Bourbaki's notion of mathematical structure as it appears in the Bourbaki's archives. Although the main ingredients of the analysis are identified early on, the exact nature of the analysis evolves considerably through the various versions of the manuscript. The principal evolution has to do with the nature of the analysis itself: first presented as a mathematical analysis, it becomes, under the influence of Claude Chevalley's contribution, a metamathematical analysis. After having presented the main ideas and lineaments, I will comment on its faith in the hands of historians of mathematics, especially Leo Corry and philosophers of mathematics. My main claim is that Bourbaki has been misunderstood and misrepresented.

David Orenstein, University of Toronto (david.orenstein@utoronto.ca), **Complementary Duality: The Bertrand Russell Archives at McMaster University and the Slater/Walsh Philosophy Collection at the University of Toronto**

Only 60 kilometres separate Hamilton's McMaster University and the University of Toronto and their respective Bertrand Russell Archives in the Williams McReady Archives and Special Collections and the John G. Salter and F. Michael Walsh Philosophy Collections at the Thomas Fisher Rare Books Library. Together they form an extraordinary resource for the History of Philosophy of Mathematics. The Russell Archives exhaustively reflect the life and work of one thinker: his publications, manuscripts and professional and personal correspondence. It includes Russell's awards, including his 1950 Nobel Literature Prize 23K gold medal and its artistically unique diploma. The correspondence also contains original correspondence from such great philosophers of mathematics as Giuseppe Peano, Louis Couturat and Ludwig Wittgenstein. It also includes correspondence between Russell and his staff and U. of T. Philosophy professor John G. Salter who declared that Russell was the philosopher who had most informed his own thought. This exchange focused on Slater's efforts to build an exhaustive collection of all Russell's publications. A selection was first exhibited in 1982, with the catalogue *Bertrand Russell: Polymath* essentially one long essay by Slater. Out of his desire to collect Russell came a massive collection of the works of modern British and American Philosophers which he has since donated tot the Fisher Library. In fact Slater has published a pair of two volume sets of Bibliographies of the British and American Philosophers respectively. Only noted (with an asterisk) are the publications not to be found at Fisher. The 5,000 works in the Walsh Collection widens the range of philosophical works geographically and chronologically such as Couturat's 1901 *La Logique de Leibniz* (Paris) or an early edition of Aristotle's *Organon.*

Jean-Charles Pelland, Université du Québec à Montréal (jcpelland@hotmail.com), **Arithmetic, Culture, and Attention**

Despite the tremendous progress made in the study of numerical cognition, an important question remains: given the precision and size limitations of our innate numerical systems, how do we manage to understand natural numbers? To explain this development, many accounts rely on culturally-inherited numerical artefacts and features of extended cognition. I argue that this externalist, culture-based approach is incomplete at best. To make my case, I detail some of the shared commitments of a few externalist accounts (e.g. De Cruz 2006, 2007, 2008; Menary 2015; Carey 2011) and highlight their reliance on cultural evolution. I then argue that externalists cannot explain the initial development of numerical content by inquisitive individuals in a numeral-free environment, since accounts of cultural evolution depend on unidentified individual-level psychological processes to explain the generation of novel content (Richerson & Boyd 2005). I illustrate the limitations of externalist accounts by appealing to a distinction between description and explanation (Simon 1998; Clark 1998), and between scaffolded and extended cognition (Sterelny 2010). I conclude by offering a few comments on why there is good reason to think that attention to quantity is a promising internalist way to explain the origin of natural numbers.

José Perez Escobar, ETH Zürich (jose.perez@gess.ethz.ch), **Mathematical Modeling in Physics, Biology and Their Intersection: Differences in the Use of Mathematical Tools across the Empirical Sciences**

Biology has been proposed to be irreducible to strictly mechanistic sciences such as physics due to its employment of certain non-mechanistic idealizations. Teleological notions, for example, play an important role in the explanations of modern biology. In the last few decades, mathematical modeling, a common mathematical tool in mechanistic sciences, has been introduced in several areas of biology with varying degrees of success, and collaborations between physicists and biologists are now more frequent than ever. The progressive success of the mathematical representation of biological phenomena can be understood as a compelling reason to consider that biology can -or must- do without the idealizations that have characterized it as an autonomous science. However, there are disparities between the proceedings of physicists and biologists. Specifically, when mathematical modeling is the link that makes these scientists come together, there are disagreements on how to build models, how to use them, and what they represent. Notably, the teleological notions that impregnate biology are introduced in mathematical models of biological phenomena in a variety of guises. While it is well known that empirical scientists and mathematicians use mathematics discordantly, and mathematical rigor is often in opposition with the practice of physics, not a lot of attention has been paid to how mathematics is used in different empirical sciences. How communities of different ideological and scientific backgrounds use mathematics to represent empirical phenomena and how they interact at the intersections of their disciplines are prerequisites for a proper epistemological understanding of mathematical modeling.

Toby Reid, University of Toronto (toby.reid@mail.utoronto.ca), **Early Pedagogy of General Relativity Theory in the USA for American Relativistic Cosmology in the 1920s and 1930s**

How an American community of relativistic cosmologists emerged in the second quarter of the twentieth century has heretofore not been investigated. This is a significant historical undertaking and necessitates answering two key pedagogical questions. How did early American relativistic cosmologists learn general relativity theory (GRT)? How, where, when, and by whom was GRT first formally taught in the United States? Addressing these questions requires primary source material. The answers will point to shortcomings in, and subsequent improvements to, advanced-level mathematics taught at American universities in the 1920s and 1930s. Preliminary research suggests no single source for formal GRT education or training, either in the USA or Europe, for pioneering American relativistic cosmologists; their GRT training was variously obtained from European mathematicians and theorists. Also suggested is that American GRT education began via graduate courses taught by cosmologist-instructors in select universities? mathematics or physics departments: for example, Caltech (under Tolman; 1922 onwards), Princeton (under Robertson; 1931 onwards), and GWU (under Gamow; in 1937). That these pedagogical developments occurred during the pre-renaissance “low water mark” (i.e. pre-1950) period of GRT is relevant. This research will elucidate early GRT education in the United States before WWII-end, a subject which is underrepresented in the published literature.

V. Frederick Rickey, United States Military Academy (fred.rickey@me.com), **Professor Bolesław Sobociński and Logic at Notre Dame**

Sobocinski (1906–1980) received his Ph.D. in 1938 under the direction of Jan Lukasiewicz (1878–1960) and then served as assistant to Stanisław Leśniewski (1886–1939). This close contact with the two founders of the Warsaw School of Logic determined the course of his research. He played an important role in the Polish underground during WW II, escaped to Brussels where he worked for several years and then emigrated to the US. After a few years in St. Paul, MN, he joined the faculty at the University of Notre Dame. He founded the *Notre Dame Journal of Formal Logic* and edited it for 19 years. We will discuss his interesting life and make some remarks about his contributions to logic.

Dirk Schlimm, McGill University (dirk.schlimm@mcgill.ca), **What was Boole's System of Logic about?**

It is commonly held that George Boole (1815-1864) invented what is nowadays called propositional or sentential logic. Here, the variables stand for propositions or sentences of which only their truth values (true/false) are of interest, and which are connected by the logical connectives "and," "or," "not," and "implication." However, while Boole's work certainly laid the groundwork for modern propositional logic, the system that the presented in his famous *An Investigation of the Laws of Thought* (1854) is quite different from the modern one. In this talk, I will present some idiosyncrasies of Boole's system of logic and discuss some of the reasons behind his approach.

Jonathan Seldin, University of Lethbridge (jonathan.seldin@uleth.ca) (with Fairouz Kamareddine, Heriot-Watt University (f.d.kamareddine@hw.ac.uk)),**Using the History of Mathematics to Teach the Foundations of Mathematical Analysis**

At a meeting of the Canadian Mathematical Society in 2005, Keith Devlin gave a talk in which he discussed what he called formal definitions, which are definitions that nobody can understand without working with them. These formal definitions are common in advanced mathematics, including analysis, but for students without mathematical maturity they are very difficult to learn from. We are in the process of writing a book, tentatively entitled A Non-Formal Introduction to Mathematical Analysis in which we are trying to avoid the use of formal definitions in this sense as much as possible. We are using two main uses of the history of mathematics: 1) to explain how proofs evolved from seeing diagrams in a certain way to becoming sequences of statements, as explained by Seldin, in his paper "Reasoning in elementary mathematic," presented to this Society in 1989, and 2) to use ancient Greek proofs using the method of exhaustion, to lead to the ϵ, δ-definition of a limit as explained by Seldin in his paper "From exhaustion to modern limit theory," presented to this Society in 1990.

James T. Smith, San Francisco State University (smith@sfsu.edu) (with Elena A. Marchisotto, California State University—Northridge (elena.marchisotto@csun.edu)), **Intermezzo**

During the 1890s, followers of Corrado Segre and Giuseppe Peano at the University of Turin sparred about rigorous foundations of projective geometry. At the nearby Royal Theater, Giacomo Puccini opened his first hit opera, *Manon Lescaut*. Two mathematicians?the southerner Federico Amodeo and Mario Pieri, a childhood neighbor of Puccini in Lucca?worked with both schools. Amodeo published first, in 1891. His paper, phrased traditionally, could suffer mild criticism for cloudiness. In 1895 Pieri began his series of papers that laid a foundation that we use today, using Peano's mathematical logic and featuring extreme precision in postulates and proofs. Pieri's innocent remark that he had proved, from his postulates, parts of Amodeo's, led to a rebuttal in Amodeo's 1896 paper on that subject. Pieri responded in print; Amodeo's 1897 reply was shrill and displayed poor logic. Finally, Pieri published a short but sharp paper whose title, *Intermezzo*, alluded to that part in Puccini's opera. Pieri lampooned and demolished Amodeo's polemic. Pieri's later papers featured deeper and broader presentation of his deductive techniques that became part of the core of modern axiomatic mathematics. I will present more details, the cultural setting, and the reaction of the German mathematical reviewers.

Valérie L. Therrien, University of Western Ontario (vtherri@uwo.ca), **The Axiom of Choice and the Road Paved by Sierpiński**

The acceptance of AC can be seen as "a turning point for mathematics (...) symptomatic of a conceptual shift in mathematics" (Kanamori 2012, 14). From 1908 until 1916, articles supporting AC or exploring some of its consequences were scant and scarcely concerted. Whilst Western Europe remained hostile to this new vision of logic and mathematics, it was at the Warsaw School of Mathematics that the seeds of this conceptual shift landed. The situation changed in 1916 when Sierpiński published a series of articles on AC and revived the dormant debate surrounding AC. Eschewing theoretical concerns about the nature and methodology of mathematical practice, he recentred the discussion towards AC's consequences, interrelations and degrees of necessity within various proofs, as well as its role in obtaining various basically trivial mathematical theorems. His programme was to eventually completely supplant the previous philosophical and methodological debates. The posterity of AC as we know it would be unimaginable without Sierpiński's efforts: "Since the labours of Mr. Sierpiński and of the Polish School, a revolution has been produced. A certain number of mathematicians have fruitfully used the axiom of choice; things are no longer in the same place" (Lebesgue 1941, 109).

Robert Thomas, University of Manitoba (robert.thomas@umanitoba.ca), **Why Mathematical Style Does not Need Philosophical Justification**

It has been correctly claimed that Errett Bishop's constructive mathematics has not been philosophically justified. This paper considers justification of such limitations on the forces deployed in various styles of mathematics and finds, for Bishop as an example, mathematical and scientific but not philosophical justification. Restrictions are considered in terms of Brian Rotman's 1993 refinement of Philip Kitcher's 1984 ideal agent, which performs mathematical operations. They are found throughout mathematical history from ancient Greece (possibly Egypt). Style is considered with an artistic analogy. It turns out that restrictions are a common feature of contemporary mathematics; instead of exploring a given landscape (an image common to G. Frege and G.H. Hardy), exploring what can be done with a specific tool like K-theory. In an inevitably pluralistic spirit, there is no philosophically based rejection of doing other things—of working in other styles. There is nothing wrong with what K-theory can't do; such restrictions are out of interest, not on principle.

Aaron Thomas-Bolduc, University of Calgary (athomasb@ucalgary.ca) (with Eamon Darnell, University of Toronto (eamon.darnell@mail.utoronto.ca)), **Strengthening Truth**

Most axiomatic theories of truth are formulated with first-order Peano arithmetic (PA) as a base theory. Although this is a good choice from a pragmatic perspective, truth over stronger theories ought to be investigated if we are hoping to describe a univalent truth predicate. Most interesting discourse makes use of more resources than PA, and this is particularly true and important for scientific and mathematical discourse where truth is central. It is first established that for any investigation aimed at a univalent notion of truth, untyped, compositional theories that prove the global reflection principle are preferable. Omega-inconsistent theories are also ruled out, as they are particularly ill-behaved in the higher-order case. Given the strength of theories we tend to care about, a move to truth over ZFC or truth in a second-order setting should be considered. The case of ZFC is briefly discussed before it is shown that truth over higher-order base theories is not only preferable for investigating a univalent truth predicate, but that such settings will provide plausible solutions to questions like that of the conservativity of truth that are of concern to deflationists.

Inna Tokar, City College of New York (innatokar@gmail.com), **History of Mathematics Education for Gifted Students in the Former Soviet Union**

This presentation will continue to examine programs for mathematically talented students in the former Soviet Union. Special emphases will be given to the boarding schools for gifted students at Moscow, Novosibirsk, St. Petersburg and Kiev Universities. The origins and history of education for gifted students in the former Soviet Union will be discussed. Specifically, the following questions will be considered: 1) Role of Khrushchev's Polytechnic reform in creation of special schools for mathematically gifted students. 2) Why and how were these schools organized? What is the nature of and variations among special school curricula and student, faculty, and alumni bodies? To answer these questions, original literature from Russia and Ukraine was reviewed, including scientific publications, educational journals, government and university documents. Interviews were conducted with Soviet-born mathematicians and educators who created and taught at these schools.

Glen Van Brummelen, Quest University (glen.vanbrummelen@questu.ca), **Jamshīd al-Kāshī's Tables of Planetary Latitudes**

Jamshīd al-Kāshī, one of the greatest human calculators of all time, composed his masterpiece of computational astronomy—the Khāqānī Zīj—in early 15th-century Iran. Within its pages we find a set of double-argument tables for determining the latitudes of the planets. The tables for the superior planets contain no entries; the table for Mercury is full; and the table for Venus is incomplete. Elsewhere in the Zīj we find a startling original method of finding latitudes, but we demonstrate that his tables do not make use of it. We provide the results of statistical and historical analyses to make what conclusions we can about how al-Kāshīcomposed these tables, partly as a case study for the power and limitations of the use of computational methods to make inferences about historical tables.

Maryam Vulis, York College—City University of New York (miryam@vulis.net), **Ukrainian Mathematicians of the 19th-20th Centuries and their Contributions to the Development of Mathematics and Mathematical Culture in Ukraine**

In the 19th century, Ukraine did not exist as an independent country; nevertheless, it had its own culture and traditions closely related to academic life. Remarkably, the first mathematical literature actually written in the Ukrainian language first appeared at the end of the 19th century. The mathematical terminology was introduced and the first mathematics high school textbooks were written in the Ukrainian language. Later, in the 1920s, some professors taught at the Lviv Underground Ukrainian University which provided education in Ukrainian and lasted for several years. It existed alongside with the famousLviv School of Mathematics associated with word-known mathematicians such as Banach and Ulam. Notably, Lviv was not even part of Ukraine until 1939, instruction was conducted in Polish, and the Ukrainian language was suppressed. In this presentation, we will discuss the impact of the Ukrainian mathematicians of that period on the development of the national science in Ukraine.

Maria Zack, Point Loma Nazarene University (mzack@pointloma.edu), **Everyone's Favorite Curve: The Cycloid**

The cycloid is a simple curve with an interesting history. Many well-known mathematicians of the seventeenth and eighteenth centuries studied the cycloid. These include Roberval, Descartes, Pascal, Wallis, Huygens, Fermat, Newton and Leibniz and a few Bernoullis. This talk will consider the work done on the cycloid by a few of these individuals and examine how their work connects to the development of some of the fundamental ideas of calculus.

Cancelled Talks

Hasan Amini, University of Tehran (hasanamini@ut.ac.ir), **Is There Any Ancient Philosophy of Mathematics?**

When it comes to the philosophy of Mathematics, as it has been presented in most introductory texts, the key roles are always being played by four famous major schools, namely fictionalism, intuitivism, formalism and Platonism, among which only the last one considered to have roots in the ancient philosophy. However, recent research in the history of ancient and medieval mathematics in general, and history of Greek and Islamic mathematics in particular, well indicates this simplified image is not realistic at all. In the light of these rather new developments, the typical portrait of history of philosophy of mathematics can be seen from a different angle. In this study, I would introduce the ideas of some ancient and medieval mathematicians and philosophers that are pivotal to altering this standard image. Furthermore, I would briefly discuss some principle questions raised during the ancient and medieval times that would fit within the context of the philosophy of the mathematics.

Henning Heller, Universität Wien (henning.heller@univie.ac.at), **Structuralism in Theory and Practice: The Case of Group Theory**

The term (mathematical) structuralism is understood both as a mathematical methodology, paradigmatically exemplified at the case of abstract group theory in the late 19th/early 20th century, and as a philosophy of mathematics developing since Benecerraf's 1965 paper. Focusing on the development of representation theory 1880-1950 (Klein, Schur, Noether, MacLane), I argue that these two roots of mathematical structuralism do not match up as well as some contemporary structuralist philosophers suggest: Firstly, the sharp distinction between a structure and its objects/positions does not transfer to mathematical practice, where groups as structures and groups as objects of broader structures are not distinguished. Secondly, the axiomatic definition of groups in terms of their internal operations is in practice much less important than groups' external operations on sets; a fact that violates the philosophical structuralists' dictum of purity of the axiomatic method. Thirdly, some elementary group-theoretic theorems (Burnside theorem, Frobenius theorem) are very hard (or impossible) to prove without the application in representation theory. Fourthly, arguable the most elaborated achievement of early group theory—group cohomology—is not only motivated, but even undefinable without using representation theory or algebraic topology. I also want to investigate whether these observations call for categorical structuralism.

Mohammad Saleh Zarepour, University of Cambridge (msz26@cam.ac.uk) **Avicenna on Infinity: Revisiting the Mapping Argument**

Avicenna believed in finitism. He argued that magnitudes and sets of ordered numbers and numbered things cannot be actually infinite. In this paper, I will discuss his main argument against the actuality of infinity: The Mapping Argument. A careful analysis of the subtleties of this argument shows that, by employing the notion of correspondence as a tool for comparing the sizes of mathematical infinities, Avicenna arrived at a very deep and insightful understanding of the notion of infinity, one that is much more modern than we might expect.