May 2003

Canadian Society for History and Philosophy of Mathematics/
Société canadienne d'histoire et de philosophie des mathématiques
Tentative Schedule for the 2003 Meeting, Dalhousie University, Halifax, Nova Scotia

Friday May 30, 2003
Life Sciences Center, Room 236

8:15 AM Welcome by Len Berggren, president of CSHPM

8:30 – 10:45 **Mathematical Boundaries: Issues in the Foundations of Mathematics [Joint with CSHPS]**

Session Chair (CSHPS): Alain Bernard (CSHPS)
(8:30) Thomas Drucker (CSHPM): *Beyond the Axioms: Plato and Brouwer as Critics of Mathematical Practice*
(8:57) Yvon Gauthier (CSHPS*): Complete Induction and Infinite Descent are not the Same. Why?*
(9:24) Janet Folina (CSHPM/CSHPS): *Bolzano and the Nature of Mathematical Proofs*
(9:51) Derek Brown (CSHPS): *Meaning and Rigorization*
(10:18) Jean-Louis Hudry(CSHPS): *Smooth Infinitesimals and Mathematical Continuity*

10:45 – 11:00 Break

11:00 – noon **Topics in Mathematics History**

Session Chair Kim Plofker
(11:00) Sorin Costreie: *Leibniz's Account of Infinity and His Philosophy of Mathematics*
(11:30) George Gheverghese Joseph: *Medieval Kerala Mathematics: The Possibility of its Transmission to Europe*

noon – 2 PM Lunch and Executive Committee Meeting

1:50 – 5:40 PM **Mathematics into Modern Times**

Session Chair Isreal Kleiner / Hardy Grant
(1:50) Leo Creedon: *Robert Murphy and the Creation of Modern Algebra*
(2:20) Francine Abeles: *Henry J.S. Smith's Work on Prime Numbers*
(2:50) Erwin Kreyszig: *Curves and Their Influence on the Development of Mathematics*
Break: 3:20 – 3:40
(3:40) Lawrence D'Antonio*: The Behā Eddīn Problem*
(4:10) Jasbir Chahal: *Euclidean Algorithm - from Euclid to Galois and Kronecker*
(4:40) Roger Godard: *Kolmogorov, 1933, and after*
(5:10) Jean-Pierre Marquis: *The History of Category Theory: The First Period 1942-1956*

Early Friday Evening: President's Reception

Saturday May 31, 2003
All but *Session B* are in Life Sciences Center, Room 338

8:00 – 11:00 AM Parallel Sessions
Session A: Psychology, Pedagogy and Technology
Session Chair: Christopher Baltus

(8:00) Robert Kalechofsky: *Metaphors and Errors*
(8:26) J.D. Phillips: *How Does It Happen?*
(8:52) Madeline Muntersbjorn: *How is Mathematics Learned?*
Break 9:20 – 9:34
(9:34) Catherine Womack: *Computer Procedures and Empiricism*
(10:00) Bill Byers: *Can a Computer Do Mathematics?*
(10:26) Dennis Lomas: *A Common Type of Mathematical Intuition*

Session B: Philosophy of Mathematics
Life Sciences Center – Oceanography Wing, Room *O*3655

Session Chair: Thomas Drucker
(8:00) Gregory Lavers: *The Vagueness and Completeness of our Ordinary Notion of Mathematical Truth*
(8:30) Steven Bland: *Not to be Ruled-Out: A Defense of Wittgenstein's Views on Rule-Following and Mathematics*
(9:00) Elana Geller: *Why Indispensability is not a Problem for Arithmetical Fictionalism*
Break 9:30 – 9:50
(9:50) David Laverty: *Tait On Abstraction*
(10:20) Diana Palmieri: *Frege and* "Epistemology"

11 AM – noon **Renaissance Mathematics**
Session Chair Jim Tattersall

(11:00) David Bellhouse: *Decoding Cardano's* Liber de Ludo Aleae
(11:30) Hardy Grant: *The Mathematics of Nicholas Cusanus*

Society Luncheon: noon – 2 PM

2 – 4:40 PM **Ancient and Islamic Mathematics [Joint session with CSHPS]**

Session Chair Daryn Lehoux
(2:00) Duncan Melville: *Poles and Walls in Mesopotamia and Egypt*
(2:30) Alain Bernard(CSHPS): *Why and How Was Proclus Commenting on Euclid?*
(3:00) Glen Van Brummelen: *Something Better than the* Elements
Break 3:30 – 3:50 PM
(3:50) Len Berggren: *Courtly Knowledge: Science and Royal Patronage*

in Tenth-Century Islam
(4:20) Edward Cohen: *The Muhammadan Calendar*

5 – 6 PM **Problems in 19th Century Periodicals**
Session Chair: Francine Abeles
(5:00) Patricia Allaire: *Probability Problems in Two Early American Mathematical Journals* [Joint work with Antonella Cupillari]
(5:30) Jim Tattersall: *Problems in the* Educational Times
[Joint work with S. McMurran and F. Coughlin]

Sunday June 1
Life Sciences Center 338

Special Session: Maritime Mathematics
8:30 – 10 AM **Maritime Mathematics: Contributed Papers**
Session Chair: Tom Archibald
(8:30) Amy Ackerberg-Hastings: *Jeremiah Day and Navigation Instruction at Yale*
(9:00) Ma Li: TBA
(9:30) Kim Plofker: *The Astrolabe in India*

Break 10 – 10:20 AM

10:20 – 11:15 **The Kenneth O. May Lecture**
Jim Bennett (Museum of the History of Science, Oxford University):
Geometry, instruments and navigation: agendas for research, 1500-1800
11:20 – 12:10 **Invited Speaker** Lesley Cormack (University of Alberta):
The Grounde of Artes: Robert Recorde and the role of the Muscovy Company in an English mathematical Renaissance
Lunch 12:10 – 1:55 PM

1:55 – 2:25 PM **General Session Contributed Paper**
Session Chair Amy Ackerberg-Hastings
(1:55) Joel Silverberg: *Higher Mathematics Education In the United States: The Role of the Academy in the Years following the War for Independence*

2:30 – 5:20 PM **Euler: Special and General Session Contributed Papers**
Session Chair: Roger Godard
(2:30) Ed Sandifer: *Euler Rows the Boat*
(3:00) Israel Kleiner: *Aspects of Euler's Number-Theoretic Work*
Break 3:30 – 3:50
(3:50) Robert Bradley: *The Curious Case of the Bird's Beak*
(4:20) John Glaus: *Leonhard Euler and His Friends*
(4:50) Christopher Baltus: *The Bernoulli-Euler Proof of the Fundamental Theorem of Algebra*

Abstracts*, 2003 *Meeting of Canadian Society for History and Philosophy of Mathematics/
Société canadienne d'histoire et de philosophie des mathématiques
[including, at the end, speakers at joint sessions with CSHPS]

Francine F. Abeles (Kean University, NJ), fabeles@kean.edu
Henry J.S. Smith's Work on Prime Numbers.
Abstract. In 1857 the Oxford mathematician Henry John Stephen Smith (1826-1883) constructed a method, one that has not been described before in the literature, to produce all the prime numbers between the last prime in a certain sequence of primes, and the square of the next prime. Using work of Adrien-Marie Legendre, and Alphonse de Polignac, Smith provided the first formula to generate these primes directly. In this paper I will discuss Legendre's and de Polignac's contributions, and how Smith used them for his result.

Patricia Allaire (CUNY, Queensborough CC), PAllaire@qcc.cuny.edu
Probability Problems in Two Early American Mathematical Journals
Abstract. Two journals, edited by Artemas Martin from Erie, PA, were published at the end of the 1800s. The Mathematical Visitor and The Mathematical Magazine presented an interesting collection of probability problems from the fields of game theory, discrete probability, number theory, and geometric probability. Several readers contributed problems and solutions that required a variety of skills, from geometry to calculus. We will consider some of the problems to illustrate not only this variety, but also the difficulty that the terminology of Martin's time presents to a modern reader.
[With Antonella Cupillary (Penn State, Erie)]

Christopher Baltus (SUNY College at Oswego), baltus@oswego.edu
The Bernoulli-Euler Proof of the Fundamental Theorem of Algebra
Abstract. This involves Niklaus Bernoulli, son of Niklaus and nephew of Johann and Jakob. From 1742 to 1745 he exchanged some 11 letters with Euler, set off by Bernoulli's paper on the sum of the reciprocal squares. Their discussion took them to the Fundamental Theorem of Algebra(FTA). Over several letters, they put together in all essential points the proof of the FTA which Euler would later present in his *Recherches sur les racines imaginaries des équations* of 1749 and, we presume, in his lecture of 1746. As time permits, the talk will indicate the appearance of the essentials of the proof and some of its shortcomings.

David R. Bellhouse (University of Western Ontario), bellhouse@stats.uwo.ca
Decoding Cardano's Liber de Ludo Aleae
Abstract. Written in about 1568, Girolamo Cardano's *Liber de Ludo Aleae* is the fullest development of the probability calculus until the official development of probability in 1654 resulting from the correspondence between Pascal and Fermat. In Cardano's book there are probability calculations for the throw of three dice and probabilistic evaluations of certain Renaissance dice games. There are also calculations for the Renaissance card game primero and for the ancient Greek and Roman games using astragali or tali (knucklebones of sheep). The book has elements of a gambling manual with warnings about various ways to cheat at cards and dice. The relevance of the *Liber de Ludo Aleae* has often been ignored (for example, Hacking's *The Emergence of Probability*) or downplayed (for example, Franklin's *The Science of Conjecture*). One reason for this is summed up by Franklin who says, "It is a confusing work; it is often not revised well enough to make the author's intention clear, and there remain in it sections explicitly contradicted by later ones." The purpose of my paper is to make Cardano's intentions clear, to remove the perceived confusion in the book and to rationalize the contradictions. The key to decoding the *Liber de Ludo Aleae*, including the probability calculations in it, is Aristotle's *Ethics*.

Len Berggren (Simon Frasier U), berggren@sfu.ca
Courtly Knowledge: Science and Royal Patronage in Tenth-Century Islam
Abstract. By the end of the tenth century C.E. the political power of the once-legendary caliphal court had been broken, and the caliph in Baghdad had been reduced to a figure-head with, at best, vestigial religious authority. Political power had devolved to a number of local courts, where kings and viziers vied with each other to attract the outstanding scientists, scholars, and litterati who created 'the tenth-century Renaissance'. Two prominent courts, those of the Buyids and the Samanids, will be the foci of this investigation into the external conditions of the production of scientific knowledge during the period.

Steven Bland (U of Western Ontario), sbland@uwo.ca
Not to be Ruled-Out: A Defense of Wittgenstein's Views on Rule-Following and Mathematics
Abstract. It is hard to deny that Wittgenstein's philosophy of mathematics has fallen into disfavor with many philosophers of late. One of the principle reasons for this recent trend is the large number of critiques which have come from the pens of influential philosophers. First and foremost among these adversaries is Michael Dummett, who accuses Wittgenstein of being a 'full-blooded conventionalist' and proceeds to criticize this position with similar arguments to those that discredited the conventionalism which had been advocated by many Logical Positivists. It is the aim of this paper to rehabilitate Wittgenstein's position by putting forward a thorough exposition of G.P. Baker and P.M.S. Hacker's interpretation of rule-following and its relation to Wittgenstein's philosophy of mathematics. An integral part of thisproject will involve the denial that Wittgenstein can rightfully be classified as a 'full-blooded conventionalist' and thus, that Dummett's criticism is without force.

Robert E. Bradley (Adelphi University), bradley@adelphi.edu>
The Curious Case of the Bird's Beak
Abstract. One of the shortest publications by Leonhard Euler in the Enestrom catalog is a brief notice (E.180) that appeared in the volume of the Memoires of Berlin for 1750, ceding priority for the solution of the problem of the precession of the equinoxes and the nutation of the earth's axis to Jean d'Alembert. At the end of this one-paragraph note is the cryptic comment: "... Mr. d'Alembert also was the first to give a resolution to the question of the nature of curves which have a cuspidal point of the second kind, or a bird's beak." We trace the development of this discovery from the Marquis de l'Hopital, who defined the cuspidal point of the second kind, through J. P. de Gua de Malves, who published a false proof in 1740 that algebraic curves could have no such cuspidal points. In the 1740s, both d'Alembert and Euler fashioned counterexamples to de Gua's claim. We examine the appearance of these counterexamples in their correspondence, and seek to determine why Euler ceded to d'Alembert on this point, although he might fairly easily have established his own priority.

Bill Byers (Concordia Univ., Montreal),wpbwork@sympatico.ca
Can a Computer Do Mathematics?
Abstract. This paper begins by discussing the relationship between computers and mathematics and by isolating aspects of that relationship that may be of interest to the philosophy of mathematics. It compares the rigor of mathematics with another property: "ambiguity." Ambiguity, as we use the term, arises in a situation in which a single situation or concept can be understood from multiple perspectives. Examples of the ubiquity of such mathematical ambiguity will be demonstrated in examples ranging from the most elementary to the proof of Fermat's Last Theorem. It is the way in which mathematics utilizes ambiguity and contradiction that differentiates mathematics, as it is done by the human mathematician, from the algorithmic "thinking" of computers.

Jasbir Chahal (Brigham Young University), jasbir@math.byu.
Euclidean Algorithm- from Euclid to Galois and Kronecker
Abstract. Even though it was Euclid who devised the Euclidean Algorithm to compute the greatest common divisor $d = (a,b)$ of two integers a and b, he did not realize that it can also be used to solve the linear Diophantine equation $ax + by = d$. This was noticed for the first time by the Indian mathematician Aryabhat (5th Century AD).He used the solution of this Diophantine equation to solve the Chinese Remainder Problem. During the 19th Century, it was further noticed by Galois and Kronecker that the same is true when a an b are polynomials Whereas, Galois used it to construct all finite fields (also called Galois fields), Kronecker's motivation was to construct a finite extension L of a given field K in which a polynomial f(x) with coefficients in K has a root. An abstract way of doing this is as the quotient, $L = K[x]/((p(x))$ of the ring K[x] of polynomials with coefficients in K by the ideal (p(x)) generated by an irreducible factor p(x) of f(x). We will explain how to view these constructions of Galois and Kronecker in a down to earth

way, as was done by Galois and Kronecker, and not in the disguised manner as is presented these days in courses on abstract algebra.

Edward L. Cohen (Ottawa), edcohen@alum.mit.edu

The Muhammadan Calendar

Abstract. The history of the Muhammadan calendar began when Muhammad made reference to it a year before he died, which occurred on 8 June 632CE. (Some say Caliph 'Umar instituted it in 639CE.) That system is still in use among the Islamic people wherever they are. What happened before among the Arabs is still debated. [Bond 1869: 228 (Handy-Book of Rules and Tables)] states: "The era of the Mohammedans, called the *Hegira* or 'Flight of the Prophet,' dates from the day on which Mohammed entered Medina after his flight from Mecca (Friday the 16th of July, 622 A.D.)." This date is the first day of 1 H from which all other days forward are counted. This lunar calendar consists of 30 years, 19 of which are regular (354 days) and 11 of which are embolismic / leap years (355 days). One would think that simple number-theoretic formulas would convert Muhammadan ↔ Julian [J] (Gregorian [G]) calendars. However, because of various aberrations, especially in 19th century literature, authors make several kinds of mistakes: e.g., (1) misprints; (2) problem that the Muhammadan day goes from sunset to sunset; (3) showing of the moon crescent for the beginning of the month; (4) J ↔ G leap years are different. These difficulties have to be overcome.

Sorin Costreie (U of Western Ontario), vcostrei@uwo.ca

Leibniz's account of infinity and his philosophy of mathematics

Abstract. Samuel Levey claims recently (1999) that Leibniz's philosophy comprises two different accounts of the *infinite* and that this dual approach constitutes a profound and insurmountable difficulty in his system. The source of this deep trouble, Levey holds, is that Leibniz endorses a *constructivist* view of mathematics, whereas his metaphysics counts as *actualistic*. Since it seems very hard to deny his *actualism* concerning infinity, the goal of the present paper is exactly to see whether Leibniz's view of mathematics could be indeed characterized as 'constructivism'. This investigation will constitute a good pretext for analyzing in parallel the intrinsic connection among the three modern 'isms' schools in the philosophy of mathematics: logicism, intuitionism and formalism.

The result of this analysis will be that it is true that Leibniz endorses a kind of *constructivism*, but one harmless and different from the constructivistic view Levey has in mind.

My personal contribution to the subject may be regarded as twofold. First, I present the 'worlds within worlds' model which seems to help us to understand better Leibniz's claim that matter is actually infinitely divided; in this new model, matter is conceived as a shrinking manifold of contiguous parts. Second, I will identify two types of 'constructivism' in mathematics, *weak* and *strong*, and I will show that Leibniz endorsed only a weak version of constructivism.

Leo Creedon (Augustana U College, Alberta), creedonleo@hotmail.com

Robert Murphy and the Creation of Modern Algebra

Abstract. The first half of the nineteenth century is noteworthy for the prominence of Irish mathematical research. Boole, Hamilton, Kelvin and Stokes come to mind. Less well known today is the work of Robert Murphy, who in his day was considered an equal to Hamilton. Robert Murphy was born into a poor family in County Cork, Ireland in 1806 and thanks to the generosity of neighbours, was sent to study in Cambridge in 1825. Murphy flourished in Cambridge, publishing influential textbooks on algebra and electricity, as well as several papers on integral equations. His health, personal and financial problems caught up with him and Murphy died in poverty from tuberculosis in 1843.

Murphy's work in integral and differential equations led him into the new field of the calculus of operations. In his 1836 paper "First Memoir on the Theory on Analytical Operations," Murphy proves many of the fundamental results in this new field. Included here is a careful study of the non-commutative ring of linear operators (although Murphy did not use these terms), eight years before Hamilton's more famous paper "On Quaternions", which gives what is today considered by many to be the first example of a non-commutative ring. Murphy studies the consequences of the non-commutativity of linear operators, including conjugates, Lie brackets and the binomial theorem for two non-commuting variables. He also studies exponents of linear operators, and the inverses and kernels of linear operators. Despite the title, there were no subsequent papers by Murphy on this subject. The results of this one paper on the calculus of operations were used by Duncan Gregory and by George Boole to study differential equations. In this paper I will examine Murphy's life and work, with emphasis on his contributions to the abstraction of analysis and algebra.

Lawrence A. D'Antonio (Ramapo College of New Jersey), ldant@ramapo.edu

The Behā Eddīn Problem

Abstract. The Islamic mathematician Behā Eddīn 'Amūlī (1547-1622) wrote an important algebra text entitled "Hulāṣat al-ḥisāb" or "The Essence of Computing". At the end of this work, Behā Eddīn poses seven problems which he claims to be unsolvable, including Fermat's Last Theorem in the cubic case. Another of these problems, now known as the Behā Eddīn problem, seeks to find all rational solutions to the pair of Diophantine equations,

[illegible] The origins of the problem lie, of course, in the work of Diophantus and the transmission of his ideas to the Islamic world. This is followed by a general consideration of the text of Behā Eddīn and its relation to algebraic developments in Europe.

Specific solutions to the problem were obtained in the 19th century, initially by Angelo Genocchi in an 1855 paper. More general methods for attacking the problem were obtained by Édouard Lucas in a paper of 1877. Lucas transforms the original problem into one of solving for rational points on an elliptic curve. In the 20th century these methods are generalized by Horst Zimmer in 1983 to give a complete description of the group of rational points on the Behā Eddīn elliptic curve. In summary, the Behā Eddīn problem provides an interesting study in the interaction of mathematical ideas in number theory, algebra, and geometry.

Thomas Drucker (University of Wisconsin—Whitewater), druckert@mail.uww.edu
Beyond the Axioms: Plato and Brouwer as Critics of Mathematical Practice
Abstract: The term 'Platonism' is used to describe a philosophical position of great contemporary interest and widespread support. The difficulty is that its connection with Plato's views of mathematics is remote. Plato was heavily influenced by aspects of mathematics but also criticized mathematical practice. By way of comparison, L.E.J. Brouwer was a practicing mathematician who also found serious flaws in the way mathematics was done early in the twentieth century. Brouwer's criticisms were regarded as threatening by some of the mathematical community, although some of Brouwer's complaints were embodied in the axiom system produced by his student Heyting. There are conflicting assessments of whether that axiom system and its consequences adequately represent Brouwer's views. Plato's views on mathematics have sometimes been embodied into accounts of truth, but they have not been turned into an axiom system. The talk will describe why Plato's views are perhaps even harder to incorporate than Brouwer's were and what mathematical practice stands to gain from keeping Plato's criticisms in mind.

Janet Folina (Macalester College), folina@Macalester.edu
Bolzano and the Nature of Mathematical Proofs
Abstract. Few mathematical pictures are as convincing as the typical graph associated with the Intermediate Zero Theorem (IZT). Bolzano and others, however, rejected such justifications as inappropriate in analysis proofs. The typical reasons then given for desiring "purely analytic" proofs is that geometry involves information that is impure or "alien" to analysis. Bolzano also urged that a proper proof should provide the real reason, or "ground", for the result; and he was concerned about circularity when proofs are not confined to their proper grounds. Interestingly, the issue of deductive rigor is not highlighted in these arguments against reliance on geometric pictures, though more rigorous definitions and proofs are now regarded as the result of this purification in analysis.
The issue of what counts as a mathematical proof remains an open philosophical question. Recent arguments against picture proofs tend to emphasize their lack of deductive rigor, the underlying assumption being that a mathematical proof must be discursive and deductively valid, or close to it. This assumption, however, is somewhat contentious. I will argue that Bolzano's somewhat different concerns about purity and proper grounds can be applied against some other picture proofs as well; and in a way that bypasses, or at least de-emphasizes, the issue of deductive rigor.

Elana Geller (U of Western Ontario U), egeller@uwo.ca
Why Indispensability is not a Problem for Arithmetical Fictionalism
Abstract. Most people feel pretty confident that they know what numbers are. Most of the time, especially when immersed in mathematical practice, one does not question their

existence. The mathematician usually takes for granted what "2" means, or she may think it trivial; "2," she may say, is 2. The issue, unfortunately, is not that simple. In this paper I will discuss two theories, Platonism and fictionalism, with the emphasis on the latter. I will present the main problems, put forward by Benacerraf, which causes problems for Platonism and out of which rises the fictionalist: the problem of criteria for mathematical truth and the problem of multiple interpretability. Fictionalism, at first blush, seems better equipped to handle the Benacerraf problems. Yet fictionalism has its own problems. The main problem for the fictionalist account is known as the Quine-Putnam objection, or indispensability, those who put forward this problem contend that mathematics, (and by extension mathematical entities), is necessary for scientific activity; and, thus, indispensable. In response Hartry Field attempts to show that scientific theories do not need numbers. I, however, argue that the fictionalist does not have to respond to the objection of indispensability. I think the fictionalist can maintain that mathematics is indispensable while still remaining a fictionalist since the thesis of indispensability does not necessitate one to take any ontological stance. I end the paper by discussing why I think fictionalism, although not naively, is commonsensical.

John S. D. Glaus (Euler Society), restinn@midmaine.com
Leonhard Euler and His Friends
Abstract. Clifford Truesdell has said, "Until the correspondence of Euler is published and studied, it will remain impossible to compose a just intellectual history of Europe in the middle of the eighteenth century."
The book "Leonhard Euler and His Friends," by Gustave-Louis Du Pasquier, is a history of the life and times of history's most celebrated mathematician. As we weave through Euler's scientific and social accomplishments we are forced to realize that Euler is one of the great representatives of the age of Reason. The story of Euler life races through the courts of Saint Petersburg and Berlin, referring to Paris on many occasions due to the Academy's enormous sphere of influence. There are vagaries through the minor academies Europe and Du Pasquier lays down the misdeeds of Euler's foes and their follies all in the highly engaging atmosphere of the royal courts of Europe This paper, albeit short in length, contextually points the reader in the exactly correct direction for further research.
Gustave-Louis Du Pasquier was a professor of mathematics at the University of Neuchatel who had nine solid titles associated to his name including editor of Volume 7 Series Prima Opera Mathematica, which had to do with recreational mathematics, probability and its applications to statistical mathematics, the theory of error and life insurance.
The paper will review the two first chapters of Du Pasquier's book, covering Euler through 1741.

Roger Godard (Royal Military College of Canada, Kingston), godard-r@rmc.ca
Kolmogorov, 1933, and After

Abstract. We analyze Kolmogorov's *The Foundations of the Theory of Probability*, published in German in 1933. Kolmogorov was 30 years old. His book is the first modern textbook in the Theory of Probability. Kolmogorov was very enthusiastic about the axiomatization of the Theory of probability, and he was strongly influenced by D. Hilbert and H. Lebesgue. We then follow the evolution of the Theory of Probability up to Doob's *Stochastic Processes* (1953).

Hardy Grant (York University) hgrant@freenet.carleton.ca
The Mathematics of Nicholas Cusanus
Abstract. The great 15th-century cardinal was drawn to mathematics as an amateur (in the original sense) and for the way in which it suggested and illumined points of his philosophy. His ensuing ventures into technical questions occupy a surprising proportion of his collected works, and evoked surprising praise ("supreme mathematical genius") from no less a historian than Josef Hofmann. I shall offer a survey of some of these investigations.

George Gheverghese Joseph (Universities of Toronto, Exeter, Manchester), george.joseph@utoronto.ca@exeter.ac.uk
Medieval Kerala Mathematics: The Possibility of its Transmission to Europe
Abstract. Mathematical techniques of great importance, involving elements of the calculus, were developed between the 14th and 16th centuries in Kerala, India. In this period Kerala was in continuous contact with the outside world, with China to the East and with Arabia to the West. Also after the pioneering voyage of Vasco da Gama in 1499, there was a direct conduit to Europe. The current state of the literature implies that, despite these communication routes, the Keralese calculus lay confined to Kerala. The paper is based on the findings of an ongoing research project, which examines the epistemology of the calculus of the Kerala school and its conjectured transmission to Europe. The paper will describe strong circumstantial evidence for the conjecture of the transmission of the calculus from India to Europe via the Jesuit missionaries. We suggest that this evidence satisfies established transmission criteria.

robert kalechofsky (Marblehead, MA), micah@micahbooks.com
Metaphors and Errors
Abstract. The view that cognitive processes are embodied as metaphors in our brains is the basis for a philosophy of mathematics and science and a psychological overview which sees mathematical and scientific ideas as metaphors and cognitive processes in our brains. This opposes the Platonic views of mathematicians who consider mathematical ideas as existing in some other realm. In addition, erring is viewed as a potentially creative process forming new metaphors (Piagetian accommodation). Paradigms substantiating this view that metaphor and error are at the roots of knowing processes (following the ideas of Piaget and Lakoff) are sketched in the work of Einstein, Cauchy and Aristotle.

Israel Kleiner (York University) kleiner@rogers.com
Aspects of Euler's Number-Theoretic Work.
Abstract. Among other topics, I will discuss Euler's contribution to what came to be known as algebraic number theory and analytic number theory.

Erwin Kreyszig (Carleton University) kreyszig@math.carleton.ca
Curves and Their Influence on the Development of Mathematics
Abstract. This paper concerns the overall role of curves in the evolution of mathematics, in particular during the time of the invention of the calculus and thereafter. In Antiquity the main effort centered around conic sections. A more general point of view was made possible by the introduction of analytic geometry by Descartes and others. In the early times of calculus, curves suggested by mechanics and geometry took the lead in the development, paving the way for the creation of a general concept of function and for general methods of differentiation and integration. This entailed the introduction of various special curves in the plane and in space named after famous (and less known) mathematicians. The creation of systematic theories of differential equations caused the development of the classical differential geometry of curves and surfaces, with curves reaching a status of their own, rather than being boundaries of surfaces, and curve theory culminating in the "natrual equations", a system of ordinary differential equations involving curvature and torsion as functions of arc length. A next milestone was the discovery of space-filling curves, challenging the notion of dimension and requiring the consideration of topological aspects, leading to increased rigor and deepening of concepts, as is reflected, for instance, in the Jordan curve theorem.

Gregory Lavers (U of Western Ontario), glavers@uwo.ca
The Vagueness and Completeness of our Ordinary notion of Mathematical Truth
Abstract. Hilbert wanted to construct a purely formal system from which all (or at least the vast majority) of classical mathematics could be derived. The purpose of this was to ensure that statements of infinitary mathematics had a perfectly clear sense. Gödel's theorem proved that Hilbert's program could not be carried out. I wish to defend Hilbert's claim that in order to have complete clarity in speaking of infinite sets we need to found such talk on finitary reasoning. If this is correct, then what Gödel's proof shows is that there is necessarily some vagueness involved in talk of such notions as 'the natural numbers'. I examine several arguments to this effect. I will then compare these arguments with some of the work by mathematical realists aimed at showing that informal theories such as second order set theory give us a characterization of the notion of mathematical truth that is complete and unambiguous. Finally I relate the above discussion to Carnap's project in *The Logical Syntax of Language*. I look at remarks that Carnap made on the subject of *LSL* after it was published that display an understanding of the above situation that is missed by modern commentators.

David Laverty (U of Western Ontario) dlaverty@uwo.ca
Tait On Abstraction
Abstract. According to Cantor, a cardinal number of some aggregate M, is "the general concept which, by means of our active faculty of thought, arises from the aggregate M when we make abstraction of the nature of its various elements m and of the order in which they are given." Since we abstract away every feature of the elements of M, the result of this double act of abstraction (the cardinal number of M), is an aggregate composed of "units". Similarly Dedekind defines numbers as "free creations of the human intellect" which result from abstracting away the particular nature of the elements of a simply infinite system. There are two basic criticisms of abstractionism due largely to Frege. First, the account is psychologistic. Numbers are treated as the product of the psychological act of abstraction. Any satisfactory account of number, however, Frege argues, should be stated in logico-mathematical terms. Second, the units of which the numbers are composed have to possess two incompatible properties. They must at the same time be identical with and distinguishable from one another. In his "Frege Versus Cantor and Dedekind On The Concept of Number", William Tait defends both Cantor and Dedekind from these Fregean criticisms. Tait argues that the accounts of Dedekind and Cantor are not psychologistic: "For neither of them are numbers psychological objects nor are the laws of number to be understood in any way as subjective." Nor is there any incoherence in their view of numbers as sets of identical units. Nonetheless, Tait argues for a version of abstraction, "logical abstraction", which stops short of defining the numbers as sets of units. Tait claims, "the idea of a cardinal as a set of pure units - call it a cardinal set - is inessential to the foundation of the theory of cardinals on logical abstraction." In this paper, I argue first that Cantor and Dedekind are much more susceptible to the charge of psychologism than Tait would like to admit. I then argue that Tait misses Frege^"s point regarding the incoherence of units being identical yet distinguishable. Finally, I offer some reasons why Tait's own version of abstraction is unsatisfactory

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A common type of mathematical intuition
Abstract: I delineate a type of mathematical intuition associated with use of some diagrams. This intuition, for convenience labelled "Aristotelean", is based on perception of these diagrams, from which abstraction takes place. "Aristotelean" intuition applies to discursive and theoretical mathematical situations and depends on (is pre-conditioned by) an understanding of these situations. Despite this dependence, this intuition is a true intuition. A couple of other preconditions for "Aristotelean" intuition are explored: the impressive capacity of visual perception seemingly instantaneously to recognize shapes and their interrelations and a significant spatial character of some types of mathematics. (The latter precondition relates to some comments of RenÈ Thom.) Examples of this intuition are drawn from absolute geometry, hyperbolic geometry, Lebesgue integration, and Fourier analysis. "Aristotelean" intuition and Kantian apriori intuition of space are contrasted. Finally, I discuss some implications for education.

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Category theory as a language: from 1942 to 1958
Abstract. In this paper, we will look at the first period of category theory, that is from Eilenberg and Mac Lane's paper in 1942 in which the first reference to functors and natural transformation is made and in 1945 which marks the official introduction of the term "category" to Daniel Kan's fundamental paper "Adjoint functors", published in 1958. We argue that before the latter paper was published and absorbed by the community, something which took a surprisingly long time, except for Freyd and Lawvere, category theory was seen as a useful language to formulate and clarify certain problems and results of algebraic topology and homological algebra. We will show how Eilenberg and Mac Lane introduced categories, what they thought it could do and what they and their colleagues did with it. Apart from a very courageous paper by Mac Lane in 1950, it is surprising to see how the theory is used by mathematicians at that time. Our analysis will stop short of the first real change in the landscape, the introduction and the usage of the notion of abelian category by Grothendieck in 1957.

Duncan Melville (St. Lawrence Univ., Canton, NY), dmelville@stlawu.edu
Poles and Walls in Mesopotamia and Egypt
Abstract. The problem of the pole leaning against a wall has a long history. In this talk we discuss the various manifestations of the problem in Mesopotamian and Egyptian mathematics and analyze some similarities and differences in presentation and solutions.

Madeline Muntersbjorn (Univ. of Toledo) utadmmunter@msgfe01.utad.utoledo.eduTitle:
How is mathematics learned?
Abstract. Being good at mathematics takes practice. Toddlers learn to count and recognize shapes. School children memorize tables, complete exercises, and check their answers against those in the back of the book. Colleges require courses in mathematics. While many grouse about it, scandal would ensue if mathematics were ever dropped from the list of requirements. Young mathematicians explore the borders of their field, eager to map uncharted terrain, while mature mathematicians train the next generation by conducting courses and composing texts. These diverse practices cohere together as integral parts of the process whereby mathematics is learned.
Philosophical accounts of this coherence vary. For example, for Plato, these activities prompt us to recollect what we once knew directly as spiritual beings, while for J. S. Mill, these activities teach us about the empirical world. This presentation will connect philosophical traditions-platonism, empiricism, logicism, etc.-with concepts drawn from contemporary education research-image schemas, manipulatives, identity modeling, etc. These connections will help us map an interdisciplinary path of inquiry towards answering the question, how is mathematics learned? In particular, I focus on the role played by mathematical symbols and representation strategies and the processes whereby

symbols are both "packed" with ever more content by experts on the mathematical frontier as well as "unpacked" by the most successful teachers and students. This speculative perspective characterizes the discovery of new mathematics and the learning of old mathematics as distinct, but inverse, processes.

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Frege and "Epistemology"

Abstract. Two schools of thought may be distinguished in terms of their respective views on knowledge. One is typically referred to as psychologism, and has as its members both Kant pre-Kantian modern thinkers. Anti-psychologism, on the other hand, appears prominently for the first time in the late 19th century with Frege's Begriffsschrift. Thinkers working within the context of psychologism take knowledge to be some subset of our true beliefs, while the anti-psychologistic school sets out to eliminate mental entities from philosophical inquiry.

Recently, proponents of psychologism (like Philip Kitcher) have advocated a reading of Frege according to which he is committed to a psychologistic account of knowledge. While anti-psychologistic thinkers tend to simply take for granted that this is wrongheaded, this paper argues and explains why Frege not only did not, but could not have adhered to a psychologistic epistemology, given his methodological principles. To show this, I highlight two (easily overlooked) Fregean accomplishments: first, he takes the view that intuition is irrelevant to explaining the applicability of arithmetic and thereby finalizes the break with Kantian tradition that had been initialized by Bolzano; second, he saw (and said) that confusing logic and psychology was a direct and avoidable hindrance to progress in the philosophy of mathematics. These factors demonstrate the novelty of Frege's work, and that novelty explains why it was successful in a way that the work of his predecessors could not have been. Psychologizing Frege by attributing to him the epistemological views of his predecessors takes away that explanatory power.

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How does it happen?

Abstract.

How does it happen that there are people who do not understand mathematics? . . . If mathematics invokes only the rules of logic, such as are accepted by all normal minds; if its evidence is based on principles common to all men, and that none could deny it without being mad, how does it come about that so many persons are here refractory? . . . That not everyone can understand mathematical reasoning when explained appears very surprising when we think of it.

—Henry Poincare, Mathematical Creation

It is instructive, but not surprising, to note that while Meno's slave boy learns about geometry and numbers with Socrates, by the time we reach Poincare such learning is no longer on the horizon of conceivability, and we are reduced simply to wondering about

what went wrong. While Plato inquired directly about the nature—the *eidos*—of number, unmediated by technique or method, by the time we reach Husserl, sedimentation—the inevitable result of the march of the scientific project to higher states of abstraction, rendering its intelligible origin forgotten, nearly inaccessible—makes this all but impossible. What went wrong?

Indeed, how *does* it happen, that most people do not understand mathematics? The Greek root of the word *mathematics* translates roughly as "that which is learnable." One can come to *know* something in mathematics—one can *learn* it—unlike, say, politics, where one simply weighs competing *opinions*. Mathematics is thus paradigmatic of learning itself; as such, the ancients understood mathematics to be a necessary preparation to philosophy (and so, on the arch to Plato's academy: 'Let no one ungeometical enter here'). And what is more human than the appetite to learn, the desire to know? How does it happen. . . Is there something about the nature of mathematics itself that renders it too difficult for most people? Or do students today resist learning mathematics—out of various psychopathologies that represent a sickness in our larger culture? Or is it just that there aren't enough Socrates' around anymore to teach them?

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Higher Mathematics EducationIn the United States:The Role of the Academy in the Years following the War for Independence

Abstract. An examination of dozens of "cyphering books" created by New England students during the 18th and early 19th centuries and other manuscripts giving evidence of their mathematical studies, together with representative textbooks published in America for the teaching of mathematics during that period, reveals a lively and widespread interest in higher education of a practical nature, including mathematical studies of quite a different type than those undertaken within the collegiate environment.

Though students often engaged in these advanced studies privately with individual tutors, these manuscripts and textbooks bear witness to the widespread influence of an institution of higher learning which has all but disappeared from the educational landscape – the Academy. First introduced to this country through the efforts of Benjamin Franklin (Penn., 1753), Lt. Governor William Dummer (Mass., 1782), and the Phillips brothers (Mass, 1778,1781), these institutions approached their calling with impressive creativity and energy. Their goal was "to induce habits of thorough and patient study, to expand and discipline the intellectual and moral powers," to provide useful and practical knowledge, "but more especially to learn them the great end and real business of living."

By 1850 ten times as many students were enrolled in academies as were enrolled in colleges and universities; and yet their very existence is largely forgotten, and they are scarcely mentioned in the histories of education that have been written since the rise of the public high school in the decades following the Civil War. We examine in some depth the nature of the mathematics taught by private tutors and Academies, from their British origins to an increasingly American view of mathematics as the goals of both the academies and the new republic matured and developed.

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Abstract: The Educational Times was first published in England in the fall of 1847. In 1861 it was adopted by the College of Preceptors as their official publication. The journal contained notices of available scholarships, lists of successful candidates on examinations given by the College, notices of vacancies for teachers and governesses, reviews, and textbook advertisements. But undoubtedly, the most important feature of this monthly journal was a section devoted to mathematical problems and their solutions. From 1864 to 1918 problems and solutions which had appeared in the journal were republished semiannually in Mathematical Problems and Their Solutions from the `Educational Times'. Many prominent European and American mathematicians contributed to the problems section of the journal. We will exhibit some interesting mathematical contributions and a few statistics gleaned from a recent classification scheme of the 18,139 problems posed in The Educational Times.
[Joint work with Shawnee McMurran, Cal State San Bernardino, and Fred Coughlin, Providence College]

Glen Van Brummelen (Bennington College),glenvb@yahoo.com
*Something Better than the _Elements_*
Abstract. Euclid's _Elements_ retained its position as the methodological standard to which geometers aspired in medieval Islam. Even so, this did not prevent several scholars from attempting to improve on it, especially the first few books. Their varied motives included logical elegance, completeness, practical concerns, and increased utility in other areas of geometry. We shall survey several of these efforts, gauge their success, and consider the extant reactions of their contemporaries.

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Computer Procedures and Empiricism
Abstract. High-speed computers have made mathematics easier to do. Obedient computational workhorses, they calculate, graph, and provide visual representations of mathematical models. Some philosophers and mathematicians worry, however, that computer-assisted proofs lack the clarity, simplicity and certainty reserved for traditional proofs. I argue that computer-assisted proofs do not represent a new and undesirable incursion of empirical methods into mathematics; rather, the problems they present highlight pre-existing epistemic challenges to standard proofs. For instance, the proof of the Four Color Theorem is too long to be checkable by any one person. It also relies on the computational authority of the computer in calculating the four-color reducibility of the cases in question. Responding to worries about surveyability, empiricism, and trust in others' results are an important part of building an adequate account of mathematical knowledge. To make my case, I present three potentially problematic models of computer proofs: computer proofs as lengthy procedure, computer proofs as arguments from