Joint Meeting of the Canadian Society for the History and Philosophy of Mathematics and the British Society for the History of Mathematics

Victoria College, University of Toronto
July 15-17, 1999

Wednesday, July 14

3:00 Tour of the Stillman Drake Collection of rare mathematical books in the Fisher Library. Meet on third floor of Old Vic, where the assembled group will proceed to the Library.

5:00-7:00 Welcome and late registration
Foyer of Emmanuel College 001.

Thursday, July 15

9:00 Conference opens
Emmanuel College 001

9:15-12:00 Morning Plenary Session, History of Applied Mathematics I
Emmanuel College 001

Chair: James Tattersall

9:15 Alexander Jones, "Mathematical Modelling and Phenomena in Ancient Astronomy"

10:15 Break; refreshments

10:30 George Molland, "Nature The Ape of Mathematics 1100-1650"

11:30 Benno van Dalen, "An Example of Applied Mathematics in Islamic Astronomy: the Calculation of the Vernal Equinox by al-Kashi"

12:00 Lunch / Executive Meeting of CSHPM Emmanuel 003

1:30-5:30 Afternoon Session A: Mathematics before 1700
Old Vic 304

Chair: Alexander Jones

1:30 Edward L. Cohen, "Calendars of the Dead-Sea-Scroll Sect"

2:00 Daryn Lehoux, "Astronomical and Civic Calendars in Roman Agriculture"

2:30 Glen Van Brummelen, "The Reconstruction of Astronomical Theories and Parameters from Medieval Horoscopes"

3:00 Break; refreshments

3:30 John Lennart Berggren, "Geometrical Analysis in Medieval Islam"

4:00 Ibrahim Garro, "Limits, Asymptotes, and Infinities Old and New"

4:30 Jacques Lefebvre, "Malebranche and Mathematics"

5:00 Sandra Visokolskis, "Newton's Theory of Infinitesimals and its Influence on Natural Philosophy"

2:00-5:30 Afternoon Session B: Philosophy of Mathematics
Old Vic 323

Chair: Alasdair Urquhart

2:00 Richard Arthur, "The Transcendality of Pi and Leibniz's Philosophy of Mathematics"

2:30 Madeline Muntersbjorn, "Imagery and Intuition in Kant's Philosophy of Mathematics"

3:00 Break; refreshments

3:30 David Laverty, "Frege and Hume's Principle"

4:00 David Boutillier, "The Problem of Mathematical Objects and Intuitionistic Logicism"

4:30 Gregory Lavers, "Set Theory's Status as Foundation for Mathematics"

5:00 Miriam Lipschutz-Yevick, "Semiotic Impediments to Formalization"

Friday, July 16

9:00-12:00 Morning Plenary Session: History of Applied Mathematics II
Emmanuel College 001

Chair: Craig Fraser

9:00 Ivo Schneider, "The Interplay Between Mechanics and Stochastic Conceptions in the Kinetic Theory of Gases of Boltzmann and his German Predecessors"

10:00 Break; refreshments

10:15 Jesper Lützen, "Mechanistic Images in Geometric Form – Hertz's Principles of Mechanics"

11:15 Break

11:30 Thomas Archibald, "Rational Mechanics and Mathematical Physics in the Crelle-Borchardt Journal, 1850-1875"

12:00 Lunch and annual meeting of CSHPM
Emmanuel College 001
Lunch provided for members of CSHPM

2:30-6:00 Afternoon Session A: History of Analysis
Old Vic 304

Chair: John Earle

2:30 Nahyan Fancy, "Analytic versus Geometric Methods for Finding Tangents to Curves"

3:00 Rüdiger Thiele, "Early Calculus of Variations and the Concept of the Function"

3:30 Break: refreshments

4:00 Robin Wilson, "The Life and Work of Edward Charles Titchmarsh"

4:30 Rebecca Adams, "The Name "Topology" from Applied Mathematics"

5:00 Gregory H. Moore, "The Influence of Klein's Erlanger Program: A Reappraisal"

5:30 Erwin Kreyszig, "On the Evolution of Engineering Mathematics"

2:30-6:00 Afternoon Session B: History of Algebra
Old Vic 323

Chair: Israel Kleiner

2:30 Muriel Seltman, "A Close Look at Some Harriot Papers"

3:00 Adrian Rice, "Extending Euler: A Little-Known Episode in the Prehistory of Quaternions"

3:30 Break; refreshments

4:00 Christopher Baltus, "Issues in the Fundamental Theorem of Algebra 1746-1816"

4:30 Francine Abeles, "Betting Round aka Pari-Mutuel Betting; A Note on C. L. Dodgson"

5:00 Hardy Grant, "Factoring Since the Dark Ages"

5:30 Jacob Appleman, "The Life of Emil Post and His 'PolyadicGroups'"

7:00 Banquet Alumni Hall Old Vic

9:00 The Mathematics of Lewis Carroll: A Dramatic Presentation
Conceived and Directed by Robin Wilson

Saturday, July 17

9:00-12:00 Morning Plenary Session: History of Applied Mathematics III
Emmanuel College 001

Chair: June Barrow-Green

9:00 Umberto Bottazzini, "Applied Mathematics: An Italian Perspective"

10:00 Break; refreshments

10:15 Amy Dahan-Dalmedico, "Models and Modelizations in Meteorology (1946-1960)"

11:15 Break

11:30 David Aubin, "Neither Pure nor Applied: Catastrophe Theory, Chaos, and the Institut des Hautes Etudes Scientifiques"

12:00 Lunch

1:30-5:00 Afternoon Session A: History of Applied Mathematics
Old Vic 304

Chair: Thomas Archibald

1:30 Robert Thomas, "What Phenomena Did Euclid Write About?"

2:00 Michael H. Millar, "Archimedes and the Concept of the Center of Gravity'

2:30 Tinne Hoff Kjeldsen, "The Origin of Nonlinear Programming: Interactions Between Pure and Applied Mathematics"

3:00 Break: refreshments

3:30 Roger Godard, "An Historical Study of the Median"

4:00 Michiyo Nakane, "Some Aspects of Mathematization in the Construction of the Hamilton-Jacobi Theory"

4:30 Steven N. Shore, "Blue Skies and Hot Piles: The Evolution of Radiative Transfer from Atmosphere to Reactors"

2:00-5:00 Afternoon Session B: History of Mathematics
Old Vic 323

Chair: Hardy Grant

2:00 June Barrow-Green, "Mathematics in Britain 1860-1940: the BRITMATH Database on the World Wide Web"

2:30 David E. Zitarelli, "An Outline of the History of Mathematics in U.S. and Canada"

3:00 Break; refreshments

3:30 Duncan Melville, "The Role of Interpretation in the History of Mathematics"

4:00 Edwardo Sebastiani Ferreira, "The Use of the History of Mathematics in Calculus Classes"

4:30 John Fossa, "An Introduction to Platonic Mathematics"

July 18

Depending on the level of interest, an excursion will be organized to Niagara Falls. On the return journey we will follow the scenic Niagara gorge from the Falls to historic Niagara-on-the-Lake, where we will stop for tea.

During and following the conference the Toronto Blue Jays will be playing at the Sky Dome. The Jays are one of the leading baseball teams of the 1990s. On July 15, 16 and 17 they will be playing the Florida Marlins; on July 18, 19 and 20 they confront the great Atlanta Braves. For details see www.bluejays.ca.

Other attractions in Toronto of interest are the Royal Ontario Museum, the Art Gallery of Ontario, the Ontario Science Centre and the Bata Shoe Museum. There are also a number of excellent heritage sights, featuring nineteenth-century history and architecture. The Metro Toronto Zoo, located north and east of the city, is one of the leading zoos in North America.

Financial support for the conference has been provided by the E. P. May Fund of the University of Toronto, the Social Sciences and Humanities Research Council of Canada and the Canadian Society for the History and Philosophy of Mathematics.

Thanks to Muna Salloum, Lydia Scratch, Bo Klintberg and John Anderson for their outstanding work as local staff for the Toronto meeting. Gordon BAKer

ABSTRACTS
Plenary Speakers

Thomas Archibald – "Rational mechanics and mathematical physics in the Crelle-Borchardt Journal, 1855-1875"

This paper will attempt to approach some aspects of the disciplinary split between mathematics and physics in nineteenth-century Germany by examining one population of authors of mathematico-physical work, namely the authors of papers on mechanical and physical subjects in the "Journal fur die reine und angewandte Mathematik" (Crelle-Borchardt) in the period between about 1855 and 1875. This work proceeds almost without exception along the lines of eighteenth-century rational mechanics. Research which incorporates the new values of precision measurement and experimental verifiability, certainly on the rise during this period, appears in other journals and (mostly) issues from the hands of other writers. In this paper we will discuss this group of writers, including Alfred Clebsch, Carl Neumann and Rudolf Lipschitz, situating their work with respect to the French tradition (represented principally by Poisson)and to Jacobi's ideas on mechanics.

David Aubin – "Neither Pure Nor Applied: Catastrophe Theory, Chaos, and the Institut des Hautes Etudes Scientifiques"

During the 1960s and the early 1970s, a few renown pure mathematicians suggested they could use some of their concepts and skills to model natural phenomena. Most of these mathematicians were specialist of the heretofore abstruse theories of topology. The creator of the catastrophe theory, French mathematician Ren Thom for the Institute des Hautes Etudes Scientifiques (IHES), near Paris, was one of their inspirational figures. This paper will describe the modeling practices of these applied topologists, and the way they were adopted and adapted by physicists in order to form essential ingredients of chaos theory. This process took place at a peculiar institution (the IHES), devoted to the pursuit of "fundamental research" while depending on the big industry for its funding. This paradoxical situation led the Institut to promote the development of general languages potentially applicable to many areas – a characterisation that no doubt applies to catastrophe theory. The most intriguing feature of the story is that the modeling practices of catastrophe theory, for which Thom mobilised an abstract philosophical discourse, were transformed by his colleague, physicist Davide Ruelle, into a new explanation for the onset of turbulence, based on the notion of "strange attractors." The paper will show the parallels between the modeling practices of "applied topologists" and the ideology of fundamental research promoted by the IHES, which responded to particular social and political contexts. Finally, the evolution of the meaning of "mathematical models" will be examined in context.

Umberto Bottazzini – "Applied Mathematics: An Italian Perspective"

Not available at time of print.

Amy Dahan-Dalmedico – "Models and Modelisations in Meteorology"

This paper will focus on scientific practices and problems of modelisations in meteorology. This domain is considered a crucial influence on the conception of mathematical models, the links with numerical instability and computers, and finally with chaos science and dynamical systems. All of these questions involve mathematics, and several mathematicians were interested in these questions.

First, I will describe briefly Von Neumann's and Charney's Meteorological Project at Princeton in the period 1946-1953 which ended with the numerical prediction per day in less than 2 hours. After this stage, the question of general circulation of the atmosphere became very important. In the late 1950s, the following alternative emerged: are the atmospheric models elaborated to understand or to predict? This debate occurred in particular during a large colloquium at Tokyo in 1960 which brought together Saltzmann, Charney, Lorenz, and others. This controversy is linked with statistical predictions versus maximum simplification of dynamic equations.

Benno van Dalen – "An Example of Applied Mathematics in Islamic Astronomy: the Calculation of the Vernal Equinox by al-Kashi"

Jamshid al-Din al-Kashi (d. 1429) was one of the most brilliant Muslim mathematicians of all times. Having grown up in Iran, he spent the last part of his life in the service of the Timurid ruler Ulugh Beg. Thus he was involved in the mathematical instruction at the religious school in Samarkand (now in Uzbekistan), in the construction of the famous astronomical observatory in that city and in the compilation of Ulugh's highly influential astronomical handbook with tables, the so-called "Zij of the Sultan."

al-Kashi is well-known for his calculation of pi to an accuracy of 17 decimal digits an a method of calculating the sine of 1 degree with arbitrary accuracy. In this paper it will be shown how al-Kashi designed some of the tables in his own astronomical handbook, the "Khaqani Zij," in such a way that values for highly complicated functions could be determined by means of only few additions and simple multiplications. We will look, in particular, at his table for determining the traditional Persian New Year, i.e., the time of the vernal equinox. Different from most of his colleagues, al-Kashi gave a table for calculating the time of the equinox precisely rather than using a constant length of the solar year. We will see that for many purposes he relied on a special type of second order interpolation which was described in many handbooks and whose use can also be shown in the "Zij of the Sultan."

Alexander Jones – "Mathematical Modeling and Phenomena in Ancient Astronomy"

The Babylonian and Ptolemaic planetary theories both rely on mathematical modeling, although the models are of quite different kinds. Babylonian planetary models are expressed in the form of computational algorithms for calculating dates and positions of certain conspicuous phenomena, and interpolation strategies for determining intermediate positions. Ptolemy's models are explicitly described as combinations of circular motions, from which numerical tables can be derived for the purpose of predicting phenomena and positions. In both systems, however, we can see that the models were not simply designed to fit the particular accepted phenomena for each planet, but also shaped by generalising assumptions that sometimes conflicted with the phenomena. The present paper will look at a few of the points under strain.

Jesper Lützen – "Mechanistic Images in Geometric Form: Hertz's Principles of Mechanics"

At the end of the nineteenth century there was a widespread belief that the phenomena of nature could and should be reduced to the laws of mechanics. However there was no agreement as to the foundations of mechanics itself. Faraday's and Maxwell's electromagnetic theory nourished the hope that one could do physics without the philosophically problematic concept of actions as a distance

Heinrich Hertz (1857-1894), whose generation of and experiments with electromagnetic waves gave strong support for Maxwell's field theory, was the first (and probably the only) physicist who wrote a mechanics book "Prinzipien der Mechanik" (1894) which did not involve either force or energy as basic concepts. Instead interactions were explained as a result of cyclic motion of a system of concealed masses connected to the observable system. Hertz argued that this was the simplest and therefore the best "image" (we would say model) of nature.

Hertz gave his mechanics a differential geometric form. In modern times he introduced a Riemannian geometry of configuration, space and geometric concepts such as curvature and "straightest path." These concepts allowed him to formulate the one law of motion that characterises his mechanics: "Every free system persists in its state of rest or of uniform motion in a straightest path." These ideas will be explained in more detail in the talk. In particular we shall have a closer look at some of Hertz's drafts of the book in order to illustrate the struggles he had to develop in a suitable differential geometric formalism.

George Molland – "Nature: The Ape of Mathematics 1100-1650"

It can be difficult to see why there should be any meaningful correlation between mathematics and the natural world, and why mathematical argumentation should be expected to increase our knowledge of it. This is by no means a new problem: it has raised its head in many ages, and various solutions have been proposed - at a least implicitly assumed, for it is not reasonable to suppose that many working mathematicians or physicists were explicitly troubled by it. In this paper I shall examine how in medieval and early modern times conceptions of nature were proposed or assumed that made it susceptible of mathematical treatment. Particular attention will be given to the Aristotelian tradition, for Aristotle both encouraged the use of mathematics in natural philosophy, and suggested that nature was often too complex to be grasped mathematically. This allowed diverse medieval emphases: for instance, Albertus Magnus was concerned to minimise its relevance, whereas Roger Bacon, Thomas Bradwardine and Nicole Oresme, while working from within an Aristotelian framework (perhaps differently interpreted in each case), made by precept and example strong cases for the legitimate applicability of mathematics to the study of nature. Among other themes, astronomy must loom large, within a tradition deriving especially from Ptolemy, but with philosophical resonances from earlier periods. Here it may be argued that perceptually the proper study of the heavenly motions had to be mathematical, and hence (though not so obviously) to conform to the elementary geometrical norms then current. With Kepler and others these norms were expanded to include more advanced forms of Greek geometry. On a terrestrial level Galileo may be seen as imposing a mathematical structure on physical reality - especially by his concentration on ideal situations. More radically Descartes proposed a new natural philosophy, which at least seemed to be isomorphic with mathematics, and hence he can be read in a strong sense as making nature the ape of mathematics.

Ivo Schneider – "The interplay between mechanical and stochastical conceptions in the kinetic theory of gases of Boltzmann and his German predecessors"

The work of Boltzmann on the kinetic theory of gases marked the end of a development that began in the late 1850s with the first contributions of Krönig and Clausius. Boltzmann also created the basic concepts like the ensembles characteristic for the successor of the kinetic theory of gases statistical mechanics. I shall concentrate on the earlier phase when the kinetic models shaped the application of statistical methods based on stochastical conceptions. Whereas the papers of Krönig and Clausius from 1856 to 1857 referred to probability theory only in order to justify the use of a mechanical model which was considerably poorer and simpler than that of a set of elastic spheres moving in a vessel, the second paper of Clausius' introduced the conception of the mean free path derived from the stochastical conception of life expectancy. After Maxwell had conceded to the model a distribution of different velocities, Boltzmann was prepared to mathematically exhaust all the properties of the mechanical models considered so far, to which he added that of a molecule with its considerably increased possibilities for different movements and configurations. By this he prepared a shift away from mechanical models to purely mathematical models on a higher level of abstraction.

ABSTRACTS
Contributed Papers:

Francine Abeles – "Betting Round AKA Pari-Mutuel Betting: A Note on C.L. Dodgson"

Charles L. Dodgson wrote two pieces on games, "Lawn Tennis Tournaments" (1883) and "The Science of Betting" (1886). His version of the "betting round" is the second item appearing in English on this betting procedure which was invented in 1865 in France by Pierre Oller. (Dodgson also wrote the first item in English, an unpublished letter.) I will describe his method and suggest some reasons for the similar themes appearing in "Lawn Tennis Tournaments" and "The Science of Betting."

Rebecca Adams – "The Name 'Topology' from Applied Mathematics"

Physicist J.B. Listing (1847) introduced the word "topology" for a study of space which considered all objects with respect to position without quantitative measure; he examined two- and three-dimensional spaces, spirals, and knots, with interest in applications to mechanics. Thirty years later P.G. Tait focused attention on Listing's work, but suggested it represented the branch of mathematics called, "Science of Position." However, the content of Listing's work became known as the first collected form of "Analysis Situs." Connections between Listing's work and twentieth century general topology will be discussed.

Jacob Appleman – "The Life of Emil Post and his 'Polyadic Groups'"

I will give the background of the life of Emil Post, a logician of the calibre of Godel and Turing. Particular emphasis will be placed on Post's work in Polyadic Groups (groups where the operation is more than binary). Though not his primary field, Post wrote a seminal work on this topic in 1940.

Richard Arthur – "The transcendality of pi and Leibniz's philosophy of mathematics"

It is usually held that the distinction between algebraic and transcendental numbers was laid down by Euler in about 1744, and that Legendre was the first to conjecture that pi might be transcendental, late in the eighteenth century (although the proof had to wait for Lindemann in 1882). So it is of some interest to see Leibniz making the same conjecture in a paper of 1676. Even more intriguingly, this occurs in the course of an analysis of the nature of magnitude that leads Leibniz to conclude that not only infinitesimals, but also mathematical figures such as circles and hyperbolae, are mere fictions or enuntiationum compendia (forms of abbreviation) – some 25 years earlier than he is supposed to have come to this position – and that continuous magnitude cannot be understood as a completed aggregate of parts.

Christopher Baltus – "Issues in the History of the Fundamental Theorem of Algebra 1746-1816"

The standard history of the Fundamental Theorem of Algebra credits Gauss with the first "satisfactory" (Stuik) or "substantial" (M. Kline) proof of the FTA, in 1799. But Gauss' proof has its own undemonstrated assumption. Is the lacuna of Gauss' proof less serious than those in proofs by d'Alembert, or Euler-Lagrange, or Laplace? (All deficiencies were resolved by 1920.) How about Gauss' proofs of 1815 and 1816? Is the question of priority the one we should be asking? This paper will be a look at the history of the FTA with these issues in mind.

June Barrow-Green – "Mathematics in Britain 1860-1940: the BRITMATH database on the World Wide Web"

The BRITMATH database is an extensive source of information relating both to the practice and practitioners of mathematics in Britain during the period 1860-1940. The data contained in the database has been gathered through systematic searches of archival sources held at the universities and other public bodies. There are biographical details of over 750 nineteenth and twentieth century British mathematicians, and details of the mathematics departments, courses and syllabi in the most important teaching institutions, as well as details of several other smaller departments. There is also information about scientific societies and journals, and a bibliography.

There is a growing interest in the history of British mathematics over this period and the database, which has very recently been adapted for use on the World Wide Web, provides a useful tool for supporting investigations into the growth of research (e.g. the schools of G.H. Hardy and H.F. Baker), the emergence of the mathematical profession, changes in the mathematical syllabus at universities, and ways in which theses aspects of mathematics were related. In this talk I shall describe the design and contents of the database, and, by giving examples of its use, explain its role in supporting research.

Len Berggren – "Geometrical Analysis in Medieval Islam"

In this paper, part of a larger study of ancient and medieval analysis written jointly with Glen Van Brummelen, I shall touch briefly on some problems in the study of geometrical analysis in ancient Greece, and then examine the development of analysis in medieval Islam as found in Ibrahim ibn Sinan's classification of problems, and defense of current practice, in his "On Analysis and Synthesis." I shall also bring out evidence from al-Kuhi and Ibn al-Haytham on the function of analysis, and close with some speculations on how the Greek "given" became the Arabic "known."

David Boutillier – "The Problem of Mathematical Objects and Intuitionistic Logicism"

Fregean Logicism treats mathematical theories as deductive systems. Some appealing features of this thesis are that it offers an explanation of the methodology, the generality, and the applicability of mathematics. However, it is well known that Fregean Logicism runs into inconsistency in its attempt to define the notion of number. Characterising the existence of mathematical objects remains important on the view that the truth or falsity of mathematical theories depends on the expressions of those theories having reference. An understanding of the reference of terms for mathematical objects is to be achieved by giving the truth-conditions for expressions containing those terms. Recently, Michael Dummett has suggested a strategy for resolving the problem of mathematical objects that involves rejecting bivalence and classical logic. In this paper I explain the developments leading up to Dummett's proposal, and then explore the prospects for intuitionist logicism.

Edward Cohen – "Calendars of the Dead-Sea-Scroll Sect"

The Dead Sea Scrolls was the name given to the documents first discovered by Bedouins in 1946 in several caves in the Qumran area, southeast of Jerusalem. They were believed to have been written by Jews called the Essenes from about 250BCE to 70CE. They were held mostly by Jordanians in east Jerusalem until the Six-Day War in 1967 and then by Israelis. However, the same group of scholars was examining them closely and it was not until the 1990s that they were open to all scholars. Nevertheless, a number of articles and books on the Dead Sea Scrolls were produced in the last fifty years – especially in the last decade. In these publications, there are a number of descriptions of what the Essenes used as calendars – particularly, the intercalated 364-day solar one and the lunar 354-day one. We try to explain the details of these.

Nahyan Fancy – "Analytic versus Geometric Methods for Finding Tangents to Curves"

The conchoid of Nicomedes was one of the curves used by the Greeks to solve the three classic problems of antiquity. Although some of the properties of this curve were studied extensively by the Greeks, its tangent was not determined until the seventeenth century. Whereas Fermat determined the tangent to the conchoid using his well-known analytic method, Barrow's construction was entirely Euclidean. Moreover, using Barrow's theorem one can construct the tangent to the conchoid of any given curve, provided the tangent to the given curve is known. The proposition of the conchoid illustrates Barrow's geometric paradigm and provides a deeper understanding of the aims of his treatise, "Lectiones Geometricae."

Eduardo Sebastiani Ferreira – "The Use of the History of Mathematics in Calculus Classes"

This paper will explore the pedagogical implications of the extremely controversial speech given by René Thom in 1984 at the Academy of Science in France entitled "The Experimental Method: a Myth of the Epistemologists (and of the Scholars?). In this speech, Thom criticises the use of the term "research method" because he believes it to be contradictory. I will explore the educational methods of using historical mathematical facts in calculus classes.

John A. Fossa – "An Introduction to Platonic Mathematics"

Two long-standing, seemingly independent, conundrums in Plato's "Republic" are those of the "Number of the Bride" and the "Number of the Tyrant." In fact, however, the two are closely related in their underlying mathematical structure. The solution of these conundrums also allows us to see how this same structure may have been used to inform other aspects of Plato's thought, such as his astronomy/astrology and his "atomic physics."

Ibrahim Garro – "Limits, Asymptotes, and Infinites Old and New"

We start with a formalisation of three paradoxes in Arabic geometry which we translated and analysed in "Logique et Analyse" vol 24, pp 351-379. The paradoxes deal with the implications of the existence of infinitesimals on the behaviour of parallel lines at infinity. We conclude that the authors of these paradoxes, al'Kindi and al'Biruni, introduced non-Euclidean concepts into their models of geometric space; which we compare with these of the hyperbolic space. Biruni's first paradox is compared with Zeno's paradoxes leading to a careful evaluation of its role in forming the geometry of space and the relativity of space and motion which are absent in Zeno.

We also look at Ibn-l-Haytham's book "Hal Shukuk Iqlidis" where the notion of infinite (indefinite) extension of a straight line is established, constructively, as a dynamical process; introducing notions and concepts usually employed in the foundations of differential geometry. This is followed by a historical critique of Ibn-l-Haytham's method of introducing motion in geometry.

We return to Kindi's paradox dealing with the extension of parallel lines (indefinitely) to infinity. We relate it to a paradox of al-Sijzi extending the constant function continuously to infinity. We compare these two paradoxes with modern results leading to their resolution.

Robert Godard – "A Historical Study of the Median"

The eighteenth century has seen the triumph of the mean value theory. Particularly, Euler, Daniel Bernoulli, and Lagrange contributed to the theory of observations. The arithmetic mean is equivalent to the centre of gravity in Mechanics. However, the arithmetic mean holds some serious disadvantages. It can be biased towards extreme values. The nineteenth century saw a long debate on the processing of outliers, and as early as 1763, the English astronomer James Short takes the mean of three quantities (see Lecoutre, 1987).

The history of the median is not as well-known. It is another measure of central tendency or the point at which a sorted sample is divided into two equal halves. Let X(1), X(2),..., X(n), be a sample arranged in increasing order of magnitude. Then the median is defined as follows:

$M = X([n+1]/2)$ if n is odd.

$M = [X(n/2) + X(n/2 + 1)]/2$ if n is even.

Therefore, the median is less influenced by extreme values. If data are symmetric, then the mean and the median coincide. Laplace, in the third edition of his "Théorie Analytiques de Probabilités" mentions very briefly the median. Even if Peter Gustav Lejeune-Dirichlet (1808-1859), published only one paper in 1836 on the Theory of Probability, Dirichlet was nonetheless a strong supporter of the median. But, it is the "English breakthrough" with Galton (1822-1911) who emphasised the importance of the median in statistics.

Herein we present two mathematical aspects of the median:

1) the distribution of probability of sorted observations.
2) the theory of sorts and the complexity of algorithms.

From a more modern aspect of numerical mathematics, the median is used as a non-linear filter, with better smoothing properties in signal processing or image processing than the arithmetic mean. Le milieu est la sagesse!

Hardy Grant – "Factoring since the Dark Ages"

In this context the "dark ages" end not around 1000 A.D. but around 1970. I shall offer a (necessarily very sketchy) overview of the main methods developed since then, showing in particular their evolution from what went before (what is arguably the "main-stream" goes back continuously to Fermat). If time allows, I shall mention one of two sociological sidelights.

Tinne Hoff Kjeldsen – "The Origin Nonlinear Programming: interactions between pure and applied mathematics"

The beginning of the mathematical theory of nonlinear programming can be dated back to the important paper "Nonlinear Programming" by Albert W. Tucker and H.W. Kuhn. Their work grew out of a project on game theory and linear programming initiated after the Second World War by the Office of Naval Research in the United States. In this talk I will concentrate on the early history of the mathematical theory of nonlinear programming. It will be seen that even though nonlinear programming originated in a context of linear programming, the driving forces behind Kuhn and Tucker's development of nonlinear programming was indeed very different from the stimulus that started the development of linear programming. I will discuss in what sense interactions between pure and applied mathematics caused the origin of nonlinear programming by Kuhn and Tucker and also influenced the beginning of duality theory for nonlinear programming.

Erwin Kreyszig – "On the Evolution of Engineering Mathematics"

In the first part of this paper we shall explore the general situation after the so-called Industrial Revolution, when applicable mathematics was available in the works of Euler, Lagrange, Fourier, and others, but most of it was not used in engineering science until the late nineteenth century.

The second part of the paper will be concerned with the development of technical colleges and universities and corresponding curricula in mathematics, initially for military purposes in France and later for the general scientific education of engineers all over Europe and North America.

In creating and successfully operating those institutions of higher technical instruction it was and is mandatory to find ways and means of teaching that adjust to the rapid development of engineering science since the turn of the century and, in particular, since the middle of the century when computers began to reach the market in larger numbers. This raises the question whether and to what extent the mathematical instruction of the engineer, and, more generally, of the applied mathematician, should pay attention to the historical development of the various subjects. In the third part of the paper we shall discuss some aspects of this problem in terms of examples.

Gregory Lavers – "Set Theory's Status as a Foundation for Mathematics"

In this paper I examine set theory's status as a foundation for mathematics. I consider a strong position put forward in a recent paper by John Mayberry. Mayberry argues that mathematics must have a foundation. This foundation may be explicit or implicit but it must be present for mathematicians to be able to understand mathematics. In this I agree with him, mathematics must have cognitive and semantic foundations for mathematics to be possible. However, he goes on to argue that an intuitive second order set theory is the only theory that could provide such a foundation for mathematics. I show that he assumes that mathematics must have a single foundation and argue that this assumption need not be accepted. I then show how, without this assumption, his arguments that mathematics is founded on intuitive set theory fail.

David Geoffrey Laverty – "Frege and Hume's Principle"

In his "Grundlagen der Arithmetik," Gottlob Frege is forced, by the Julius Caesar problem, to utilise his explicit definition of the cardinality operator in order to establish the truth of his contextual definition, now known as Hume's principle (HP). The explicit definition, however, relies on Frege's theory of extensions, shown to be inconsistent by Russell. Consequently, for may years Frege's logicist program was viewed as a failure. However, in 1983, in his "Frege's Conception of Numbers as Objects," Crispin Wright argued that one can salvage a consistent fragment of Frege's system. In the second-order theory whose sole non-logical axiom is HP, we can achieve the important result of deriving the Dedekind infinity of the natural numbers. The extent to which this result, now known as Frege's theorem, can be used as the basis for a Fregean logicism, has subsequently become a central area of research within the philosophy of mathematics. One of the main problems within this area of inquiry is, of course, the status of HP. Given that the explicit definition led to the inconsistency of Frege's theory, Wright does not have it at his disposal. However, is it sufficient for Wright's purposes to show that HP is satisfiable? Although it is true that HP has been shown to be true in all models whose domain contains the natural numbers, this result simply allows us to conclude that the conditional IF there are numbers, then HP holds is true, a result which is hardly satisfactory as a foundation for arithmetic.

Jacques Lefebvre – "Malebranche and Mathematics"

Nicolas Malebranche (1638-1715) is better known as a philosopher in the Cartesian movement. However he also worked and wrote about religion, morality, science and mathematics. He main philosophical publication "De la recherche de la verit," went through several enlarged editions as well as translations.

We shall describe the function and value he assigned to mathematics in "De la recherche." We shall also compare his thoughts on the use of mathematics in the search for knowledge and in the training of the mind to the views held by other thinkers like Descartes, Pascal, Arnauld and Nicole, and Spinoza. Lastly we shall look briefly at Malebranche's personal activity in mathematics and at the role he played in the mastery and the diffusion of the new mathematics of the day, first along Cartesian lines and then with the Leibnizian differential and integral calculus.

Daryn Lehoux – "Astronomical and Civil Calendars in Roman Agriculture"

Roman agricultural works show a strange combination of dating systems, where planting, ploughing and manuring are timed according to either the annual risings and settings of the fixed stars, the phases of the moon, or the Roman civil calendar. This paper will flesh out the convergences and divergences of these dating systems, arguing that the lunar dates are really unrelated to the calendrical and stellar dates, and that these latter two are largely interchangeable. The stellar dates, however, may date from an earlier period, and have then been adapted to fit the changing Roman luni-solar calendar.

Miriam Lipschutz-Yevick – "Semiotic Impediments to Formalisations"

Hilbert considered recognisability and concatenation of mathematical objects as intuitively given. Curry defined two kinds of formal systems: syntactical and ob.. Formal objects are objectified as linear series of signs in the first and as tree diagrams in the second. A similar duality in viewing the objects of a formal system is held in general: (1) as tokens of some physically concrete types recognized by their form (shapely objects), e.g., Kleene, marks on paper; (2) as entities of some abstract structure generated from some arbitrary zero entities, e.g. Kleene, generalized arithmetic. We inquire into the relation between the sign and that which it signifies as it pertains to the objects of a formal system. Since the formal objects as defined in the two modes exist only by virtue of their mode of definition, "corresponding" objects (objects named alike), say formal numerals (the numeral for the natural number n) in the two modes cannot be identified ab initio as being the same. We show – contrary to Curry – that e.g. formal numerals as viewed in these two modes are <u>not</u> instantiations of each other. Since the procedure used in the arithmetisation of the formal objects ("slapping together" or via a "structural buildup") mirrors their makeup, the g numberings in the two modes do not "mesh." When these modes are mixed in a metamathematical argument as, for instance, in def. 16, Godel, a fundamental ambiguity arises and the g numbering may be thrown out of kilter. Yet both modes are needed in the metamathematics. We conclude that in formulating elementary number theory in terms of symbols which stand for nothing, we must confront the meaning (nature) of these signs <u>per se</u> (as signs). This creates a big gap between what formalisation pretends to do and what is actually possible. We speculate that the recognition of abstract objects viewed respectively by form and by structure as in these two modes, reflects our knowledge of objects by perception and by description respectively.

Duncan Melville – "The Role of Interpretation in the History of Mathematics"

There seems to be a widespread conception that the role of the historian of mathematics is to uncover the truth of mathematical history and present what is ideally a correct narrative account of names, dates and theorems. In this view, debate between historians is confined to philological disputes and those tiresome arguments over priority that have given the discipline such a bad name.

In contrast, I will argue that the function of the historian is not merely the recovery of last facts, but, more importantly, the interpretation of those "facts." Constructing historical meaning from a collection of data opens legitimate areas of serious debate among historians. This aspect of the historian's craft is quite alien to the methodology of the mathematicians who are typically the teachers of history of mathematics, and to the students who study it. Presenting some of these scholarly debates can help our students become more sophisticated historians. I will give some examples of such debates by way of illustration.

Michael Millar – "Archimedes and the Concept of the Centre of Gravity"

Archimedes was the first great applied mathematician. We can see this in his work "On the Equilibrium of Planes: Book I," where Archimedes begins by postulating seven properties pertaining to the centres of gravity of certain planar figures. Then, in a remarkable sequence of fifteen propositions, he shows how to use these postulates to arrive at a determination of the centres of gravity of triangular and trapezoidal figures. We will look at a select few of these propositions to see the power of the axiomatic method at work in the hands of a master mathematical craftsman.

Gregory Moore – "The Influence of Klein's Erlanger Programme: A Reappraisal"

In 1925, not long after Felix Klein's death, Richard Courant praised Kein's Erlanger Programme as "perhaps the most influential and most widely read mathematical work of the last 60 years." Similar praise for the Erlanger programme (or EP) has continued up to the present. Yet there are remarkably few citations of EP before 1915, when Klein revived it to treat the theory of relativity. We discuss those citations, primarily by the Italian geometer Corrado Segre.

Even after 1915, EP was mainly cited as a "great work," not as a work actually being used in research. The earliest article to use EP in detail appeared in 1916 by Young and Morgan, a work we discuss at length.

Ironically, EP was inadequate even when it was originally formulated, for it could not encompass Riemannian geometry. Circa 1925, Carton and Stoutness generalised EP so as to include Riemannian geometry.

Finally, we reflect on the apparent contradiction between EP's lack of influence on research with its fame as a major accomplishment.

Madeline Muntersbjorn – "Imagery and Intuition in Kant's Philosophy of Mathematics"

Immanuel Kant (1724-1804) argued that mathematicians generate knowledge that is genuinely new, necessarily true, and does not require experience of the outside world. In his terminology, mathematical knowledge is both synthetic and a priori. The development of non-Euclidean geometries in the early nineteenth century cast doubt on Kant's claims. These alternative geometries suggested that Euclidean results were not necessary, but were merely formulations of possible relations between particular kinds of objects which need not even exist. The search for more secure mathematical moorings led scholars away from the extended magnitudes of geometry and towards the logical behaviour of numbers. The development of predicate logic in the late nineteenth century seemed to seal Kant's fate as a brilliant, but largely mistaken, philosopher of mathematics. Recently, scholars have begun to sift through Kant's voluminous writings in an effort to identify viable insights into the nature of mathematics which transcend his eighteenth century perspective on the state of the art. Kant's most important insight is his insistence that visual imagery is an essential part of mathematical reasoning. The generation of new and necessary knowledge requires intentional engagement with diagrams and symbols. These symbols are usually concrete objects in the world, but need not be. For, in either case, mathematical signs are only seen as such with our mind's eye, or "sensible intuition." Contra Frege and many other scholars, the essential role of sensible intuition is not peculiar to geometric or spatial reasoning, nor can it be rules out of mathematical practice by employing sophisticated algebraic tools of inferential analysis. Compared with geometry, algebra involves both increased freedom in our choice of signs and increased indeterminacy of the signified. But, as I hope to illustrate with examples from the history of algebra, neither this freedom not this indeterminacy is evidence for the claim that visual imagery is wholly eliminable. Contra Friedman (1992), Kant's philosophy of mathematics is more than a model of fruitful interaction between philosophy and science unique to a pre-Fregean logical context. When the interplay between visual imagery and intuition is restored to its original position of prominence, Kant's philosophy of mathematics ceases to be naïve and becomes a compelling reminder that both philosophers and historians of mathematics must attend to the interplay between form and content in the growth of mathematical knowledge.

Michiyo Nakane – "Some Aspects of Mathematisation in the Construction of the Hamilton-Jacobi Theory"

This paper examines how Hamilton-Jacobi theory in the calculus of variation was constructed. The process can be divided into two parts. The first was Hamilton's development of a new dynamical theory published in 1834 and 1835. Since his dynamical theory was derived from the analogy of mathematical forms which appeared in his optics, I examine his mathematical work in optics. The second was Jacobi's derivation of a mathematical theory from Hamilton's dynamical work. Although historians have tended to neglect this development, Jacobi's generalisation, which I propose to call a second kind of mathematisation, was necessary to establish the Hamilton-Jacobi theory in the calculus of variations.

Adrian Rice – "Extending Euler: A Little-Known Episode in the Prehistory of Quaternions"

In the famous paper written in 1749, Euler had apparently resolved the contentious issue of the existence and justification of logarithms of negative and imaginary numbers. However, some eighty years later, an unknown Irish mathematician by the name of John Thomas Graves published a paper in which he generalised Euler's result. His controversial proof re-opened the discussion (in Britain, at least) on the nature of imaginary logarithms, dividing the British mathematical community into those who supported and opposed Graves' conclusion. Although the debate which surrounded his result was relatively short-lived, this paper argues that it is of previously unrecognised significance in the history of mathematics, due to the little-known fact that one of its ultimate consequences was William Rowan Hamilton's discovery of quaternions in 1843.

Muriel Seltman – "A Close Look at some Harriot Papers"

Thomas Harriot was hailed by his contemporaries as having raised mathematics, and algebra in particular, to previously-unknown heights. A close study of his unpublished papers reveals them as surprisingly modern, both in notation and methodology. There are, though, some differences in procedure which are conceptually suggestive

I propose a detailed look at some pages which will indicate why his contemporaries though so much of him, despite the fact that he published no mathematical work in his own lifetime.

Steven J. Shore – "Blue Sky and Hot Piles: The Evolution of Radiative Transfer from Atmospheres to Reactors"

Radiative transfer theory has mirrored many of the trends characterising the development of applied mathematics during the past century. It began with the development of the phenomenological equation of transfer at the start of the century, designed to treat the passage of light through foggy opaque atmospheres. After the 1920s, the theory had progressed to the detailed modeling of stellar and planetary atmospheres and transformed into the remote sensing problem of remote diagnoses of physical conditions and abundance's in such environments. During the 1940s, the need to treat neutron transfer in complex geometries led to an interest in applying these methods to nuclear reactors, using methods of invariant imbedding, Monte Carlo simulations, and integral equations, all areas of continuing study. The formative period, mainly analytic in thrust, ended by the mid-1960s with the comprehensive treatises by Chandrasekhar, Ambartsumian, Korganoff, Busbridge, Sobolev, Davidson, and the 11th AMS Applied Mathematics Symposium on nuclear reactors. Invariant imbedding and doubling methods have been widely developed for scattering problems, mainly in planetary atmospheres and nebular transfer, while improvements in computational methods over the past 30 years have introduced advances in multidimensional, multigroup transfer codes for neutron physics (Boltzmann equation solvers), neutrinos, and significant advances in radiative transfer, and the growth of the field of radiation hydrodynamics.

Rüdiger Thiele – "Early calculus of variations and the concept of function"

Euler's famous textbook "Introductio" (published in 1748) established to concept of a function at the centre of a new branch of mathematics, later called Analysis. A lifespan earlier in l'Hospital's great textbook (published in 1696) this concept is not to be found. Up to the end of the seventeenth century mathematicians had used geometrical constructions and curves which belonged to that method instead of using functional relations. Some constructions that produced so-called mechanical curves were excluded. However, the infinitesimal calculus unified these isolated kinds of curves by the new concept of function. Firstly the modern idea appeared in problem of the calculus of variations posed by Jakob Bernoulli in 1697. Although Johann Bernoulli failed to solve the problem completely, he became aware of the implicit extension of the old geometrical concept. Moreover, he laid stress on that extension coming so to his well-known 1718 definition of a function. Focusing on essential translations I will comment on the relevant period of this development between both the cornerstones of l'Hospital and Euler.

Robert Thomas – "What phenomena did Euclid write about?"

Everyone knows that Euclid wrote a treatise called "Optics." Fewer know that he wrote another applied book, "Phaenomena." Heath dismisses it in his "History of Greek Mathematics" and anthologises its introduction in his "Greek Astronomy." What is it actually about? This paper will be an introductory look at the subject matter of this and similar texts.

Glen Van Brummelen – "Horoscopes and the History of Astronomy: Casting for Insights"

In ancient and medieval times, astronomy and astrology were inextricably linked together. Astronomical works were often constructed with the casting of horoscopes in mind; conversely, the astrologers required astronomical treatises to help determine celestial positions. The primary literature is awash with historical horoscopes containing (among other things) predictions of planetary positions. It is possible to retrieve important information relevant to the history of astronomy with elementary statistical analysis of these horoscopes. I shall present the results of a first attempt to do so, with the goal of an explanation of the astronomical system underlying the planetary predictions in the Kitab al-Kamil, an astrological world-history composed by the tenth century Muslim astrologer Musa ibn Nawbakht.

Sandra Visokolskis – "Newton's Theory of Infinitesimals and its Influence on Natural Philosophy"

The aim of this paper is to explore Newton's philosophical approach to mathematics and its incidence in the development of physical and astronomical matters. More specifically, the emphasis will be given to his point of view about infinitesimals as a tool for the "improvement of Natural Philosophy," that had peculiar connotations provided by the incorporation of mathematical techniques that nothing had to do with the typical position that colleagues of his time supported, mostly related to the search of hypothetical causes in Nature. Instead of that, it will be argued that his investigations focused "mathematically" on phenomena, and it will be shown how this research contributed to capture the general principles that governed then. In order to account for this perspective, the strategy to follow in this work will be:

a) to re-examine Hintikka's interpretation of the notions of "induction" and of "method of analysis" he though Newton assumed; and to argue that, underlying this position, is a conception of Mathematics that differed from Newton's purpose of eliminating hypotheses from scientific explanations'

b) to present a classification of Newton's processes of acquisition of mathematical knowledge in three levels, constructed to guide the suggested interpretation of his philosophical approach concerning mathematics.

Robin Wilson – "The life and work of Edward Charles Titchmarsh"

E.C. Titchmarsh was born 100 years ago, on 1 June 1899. In this talk I shall outline his work in Oxford, London and Liverpool, from his time as a research student of G.H. Hardy to his achievements as Hardy's successor in the Savilian Chair of Geometry at Oxford. I shall also briefly outline his contributions to the theory of functions, number theory, and Fourier analysis, with particular reference to his textbook writing.

David E. Zitarelli – "An Outline of the History of Mathematics in the U.S. and Canada"

This talk proposes an outline of the history of mathematics in the United States and Canada that could serve as the basis for an undergraduate course based on developments in North America from early explorations through the year 1950. Suggestions for a textbook and related readings are given, based on the experience of designing and teaching such a course. The outline of the history comes in two parts, one chronological, the other dealing with issues that arise while following a chronological order.