CSHPM / SCHPM

ANNUAL MEETING 1997

MEMORIAL UNIVERSITY, ST. JOHN'S, NEWFOUNDLAND

JUNE 7 - 8, 1997

PROGRAM

Saturday, June 7

9:00 a.m. Rebecca Adams (Southern California College)
"Mathematics and Phyllotaxis"
(paper presented by Matt Corrigan, Southern California College)

9:30 a.m. Elaine Landry (University of Western Ontario)
"Varieties of Realism"

10:00 a.m. Michael Pool (University of Western Ontario)
"Descartes' Early Philosophy of Mathematics and Perception"

10:30 a.m. Break

11:00 a.m. Aditi Gowri (University of Texas -- Austin)
"Mathematical Revitalizations: A Model for the Social Historical Study of Foundations"

11:30 a.m. Israel Kleiner (York University)
"Proof: A Many-Splendoured Thing"

12:10 p.m. Lunch / Executive Committee Meeting

1:30 p.m. Siegfried Thomeier (Memorial University)
"C.F. Gauss: Mathematician, Scientist and Inventor"

2:00 p.m. Barry Davies (SoftLogic Solutions, Inc.)
"Looking at Geodesic Sections in Curved 3-Space"

2:30 p.m. Alan Baker (Princeton University)
"Aspects of Indispensability: Infinitesimals and Quaternions"

3:00 p.m. Break

3:30 p.m. Barbara Bohannon (Hofstra University) and Sharon Kunoff (Long Island University)
"Advances in the Study of the Three-Body Problem"

4:00 p.m. Erwin Kreyszig (Carleton Uniersity)
"Some Historical Roots of Modern Numerical Analysis"

4:30 p.m. Roger Godard (Royal Military College of Canada)
"History of the Least-Squares Method and its Links with Linear Algebra"

Sunday, June 8

9:00 a.m. Amy Ackerberg-Hastings (Iowa State University)
"How Straight Must a Straight Line be to be Straight?: The Teaching of Euclidean Geometry in the Early Republican United States"

9:30 a.m. Francine Abeles (Kean College of New Jersey)
"Dodgson and Determinants"

10:00 a.m. Bill Anglin (University of Toronto)
"The History of the Bachet Equation: From Diophantus to Baker"

10:30 a.m. Break

11:00 a.m. Rudiger Thiele (University of Leipzig)
"The Mathematics and Science of Leonhard Euler"
(invited speaker)

12:00 p.m. Lunch / Annual General Meeting of the Society

2:00 p.m. John Dawson (Pennsylvania State University -- York)
"The Papers of Oskar Morgenstern: A Resource for the History of 20th-Century Mathematics"

2:30 p.m. Edward Cohen (University of Ottawa)
"A Last Attempt at Calendar Reform"

3:00 p.m. Jacques Lefebvre (Université du Québec à Montréal)
"The Importance and Ambiguity of Mathematics in the Works of Robert Musil"

3:30 p.m. Break

4:00 p.m. Hardy Grant (York University)
"M_{191} and the History of Factoring"

4:30 p.m. Michael Millar (University of Northern Iowa)
"Newton's Segmental Neusis: Construction of the Regular Heptagon"

5:00 p.m. Peter Griffiths (independent scholar)
"The Mathematics of the Projection Maps of Etzlaub (1511) and Mercator (1569), including Comments from Edward Wright (1599)"

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ABSTRACTS OF PAPERS

INVITED SPEAKER:

Rudiger Thiele (University of Leipzig)
(Sunday, 11 a.m.)

"The Mathematics and Science of Leonhard Euler"

The paper is divided into three parts. In the first we examine the life of Euler, in the second we consider his work on the function concept and his use of infinitesimals (comparing Euler's treatment to such earlier and later figures as the Bernoullis and Weierstrass). In the third part we look in more detail at a particular example, Euler's results on trigonometric functions and the sum of reciprocal squares. A larger theme of the paper concerns the relations of mathematics and science in the age of Enlightenment.

CONTRIBUTED PAPERS:

Fran Abeles (Kean College of New Jersey)
(Sunday, 9:30 a.m.)

"Dodgson and Determinants"

A set of seven unpublished letters written between 1866 and 1867 provides evidence that C.L. Dodgson (Lewis Carroll) was in the forefront of writers of texts on determinants. The letters show he had direct contact with William Spottiswoode and Bartholomew Price, and indirect contact (through Spottiswoode) with James Joseph Sylvester. Dodgson's book *An Elementary Treatise on Determinants*, published in December 1867, is among the earliest significant books written on the topic (along with those by Spottiswoode (1851), Francesco Brioschi (1854), Richard Baltzer (1857), George Salmon (1859), and Isaac Todhunter (1861)). Characterized by logical exactness, Dodgson's book contains new theorems as well as extensions of existing material.

Amy Ackerberg-Hastings (Iowa State University)
(Sunday, 9 a.m.)

"How Straight Must a Straight Line Be to Be Straight?: The Teaching of Euclidean Geometry in the Early Republican United States"

The story of geometry education in the early United States in the early 19th century has previously been told only in piecemeal fashion. This paper unifies those scattered bits of information and goes on to consider a number of questions. For example, where did students study geometry? What did they learn? How deeply did they understand the philosophical implications of Euclidean geometry? The American college mathematics curriculum was then based upon British and French textbooks. Thus the paper also forms part of a larger inquiry into the importance of geometry textbooks by Robert Simpson, John Playfair, and Adrien-Marie Legendre.

Rebecca Adams (Southern California College)
(paper presented by **Matt Corrigan** (Southern California College))
(Saturday, 9 a.m.)

"Mathematics and Phyllotaxis"

The sequence suggested by the rabbit problem in *Liber Abaci* (1202) by Leonardo Fibonacci (Leonardo of Pisa) has been related to phyllotaxis. The phyllotactic systems showing spirality belong to Fibonacci-type sequences of integers. Observational phyllotaxis goes back to Bonnet (1754); mathematical phyllotaxis begins with Braun (1831), Schimper (1836), and Bravais and Bravais (1837). This same plant structutre phenomenon has been studied by an architect, Hermant (1946), using a topological deformation of the fractal hierarchy. This discussiom represents a preliminary study suggested by the question: is there a connection bewteen the fractal geometric study of phyllotaxis and the application of the Fibonacci sequence to phyllotaxis?

Bill Anglin (University of Toronto)
(Sunday, 10 a.m.)

"The History of the Bachet Equation: From Diophantus to Baker"

The Bachet equation is the Diophantine equation $y^2 = x^3 - k$, where k is a given non-zero integer. The first complete solution was given by Alan Baker in 1968, but it was introduced as long ago as Diophantus himself. in this paper we give its history, with emphasis on the contributions of Bachet, Fermat, Mordell, and of course Baker.

Alan Baker (Princeton University)
(Saturday, 2:30 p.m.)

"Aspects of Indispensability: Infinitesimals and Quaternions"

A central task for any adequate philosophy of mathematics is to give an account of mathematical truth. One popular approach is to link the truth of mathematics to the truth of some other body of knowledge, typically logic or empirical science. With the apparent failure of the logicist program, attention has shifted to the latter of these two options. The so-called "Indispensability Argument" claims that we ought to believe in the truth of mathematics because mathematics is an indispensable part of our best scientific theories.

One problem with this argument is in clarifying what it is for a piece of mathematics to be "indispensable" for science. A second problem is that it seems to justify only those parts of mathematics that do play a substantive role in science. My aim in this paper is to address these two problems via a consideration of two historical examples, infinitesials and quaternions. Infinitesimals were superseded by the Weierstrassian theory of limits but have recently been resurrected in the guise of non-standard analysis. Quaternions were superseded by vector calculus but have recently found a role in the foundations of quantum mechanics. My aim is to use these two examples to highlight different aspects of the concept of indispensability, and to show how the Indispensability Argument might be applied to specific mathematical theories.

Barbara Bohannon (Hofstra University) and **Sharon Kunoff** (Long Island University)
(Saturday, 3:30 p.m.)

"Advances in the Study of the Three-Body Problem"

The study of the three-body problem spans the centuries. In this talk we will look at some of the different approaches taken in the study of the problem, from the quantitative methods proposed by Newton and Lagrange in the late 17th and early 18th centuries to the qualitative approach first formulated by Poincaré in the late 19th century. We also consider the work of G.D. Birkhoff in the first quarter of the 20th century and the methods used by J.K. Moser in the mid-20th century. We conclude with an analysis of where things stand today when the problem is considered a basic example in chaos theory.

Edward Cohen (University of Ottawa)
(Sunday, 2:30 p.m.)

"A Last Attempt at Calndar Reform"

Calendars are fascinating. Gauss wrote about the Hebrew, Julian, Gregorian and Easter calendars. In his *Werke* these articles are considered as astronomy, perhaps considered as a branch of mathematics in his time. Besides the calculational aspects of calendars, any of these will of necessity have to consider the solar, lunar and/or sidereal (star) phases of the subject. Unfortunately, the earth does not go around the sun in a "considerate" number of days, but rather approximately 365.2422 days. The moon goes around the earth in 29 days and an "inconsiderate" number of hours, minutes and seconds. In other words, one had to think up a calendar (Gregorian at present) that is workable but certainly not perfect. Various people tried to reform the Gregorian calendar from the 1920s to the 1950s through the League of Nations and the United Nations. Two of the many proposals by George Eastman and Elisabeth Achelis almost succeeded. Their good and bad aspects will be shown. Other proposals have recently been examined, especially one worked on very meticulously by a Dominican sister until her recent death in 1991. This calendar will never be popular, but I suggest modifications which might be acceptable.

Barry Davies (SoftLogic Solutions Inc.)
(Saturday, 2 p.m.)

"Looking at Geodesic Sections in a Curved 3-Space"

The spatial part of Schwarzschild's solution to Einstein's field equations is a three-dimensional curved space. This talk will describe an effort to illustrate curvature in this space as an application of Tullio Levi-Civita's Absolute Differential Calculus. At a given point, the geodesic section normal to a given direction is the locus of all geodesic lines whose initial direction is normal to the given direction. Using his concept of parallelism, Levi-Civita derives Ricci's equation for the Rieannian (Gaussian) curvature of a geodesic section. I will present a simple scheme to visualize these sections. In the course of the talk, I will attempt to convey how much can be added to modern engineering by looking at the history of tensor calculus. In this effort I am looking at a very important paper by Christoffel, "Über die Transformation der homogenen Differentialausdrucke zweiten Grades", *Crelles Journal* 70 (1869).

John Dawson (Pennsylvania State University -- York)
(Sunday, 2 p.m.)

"The Papers of Oskar Morgenstern: A Resource for the History of Twentieth-Century Mathematics"

The economist Oskar Morgenstern (1902-77) is best known to mathematicians as the co-author (with John von Neumann) of *The Theory of Games and Economic Behavior*. A cosmopolitan figure who moved bewteen the intellectual communitites in Vienna and Princeton, Morgenstern was associated with the Vienna Circle during his university years, and through it came into contact with Karl Menger, Kurt Gödel, Rudolf Carnap, and Abraham Wald. After his emigration to the U.S. in 1938, he developed close personal relationships with von Neumann and Gödel, as well as Hermann Weyl, Marston Morse, C.L. Siegel, and the physicists Einstein, Bohr, Pauli, Dirac and Wigner.

Morgenstern's papers, including a candid series of diaries spanning some sixty years, are available to scholars at the Perkins Memorial Library of Duke University. The speaker will survey the contents of those papers and exhibit extracts from some items of mathematical interest.

Roger Godard (Royal Military College of Canada)
(Saturday, 4:30 p.m.)

"History of the Least Squares Method and its Links with Linear Algebra"

The algebraic approach to the linear least-squares problem is the following. Let A be an mxn matrix, $m \geq n$, such that the equation $A\beta=0$ has no non-zero solution, i.e. the only acceptable solution is the trivial solution. There is a unique vector $\beta^* \in R^a$ yield-ing a minimum for $|A\beta-Y|^2{}_2$, and satisfying the normal equations:

$$A^tA\beta^*=A^tY, \text{ or } \beta^*=(A^tA)^{-1}A^tY \qquad (1)$$

In the theory of probability, the random variables are often called $y_1, y_2, \ldots, y_n$, and in statistics we give the same name to the obsevations. In order to avoid any confusion, we shall write the vector solution β^*. The algebraic approach corresponds to a "reasonable" solution of an overdetermined system of linear equations, i.e. a problem of applied linear algebra. The method of least squares in statistics is related to the theory of the accidental errors ϵ, linked to the observations. We put

$$A\beta+\epsilon=Y \qquad (2)$$

where Y are the observations and β^* is the corresponding least squares estimate of the vector parameter β. More generally, given a model function $f(\beta,X)$ where β is the variable of the n parameters, and m observations y_i, the optimization problem is:

$$\min S(\beta) = 1/2\ \Sigma(f(\beta,x_i)-y_i)^2 \qquad (3)$$

The history and solution of the least squares problem is rich and multidisciplinary. It corresponds to a problem of approximation which for a long time has been mixed with a problem of interpolation. The second aspect, initialized by Gauss, is probabilistic, and it is linked to the method of the maximum likelihood. Finally, numerical mathematics has introduced new theorems, new algorithms, new concepts to the above problem.

The classical least squares problem corresponds to a problem of mathematical modelling that we call "the hypothesis" in statistics. This model is given a priori and we must find the estimators to parameters of a given model by using the data, the observations, obtained a posteriori by minimizing the sum of errors squared with respect to the model.

We shall try to follow the problem of optimization, a problem of minimization up to the axiomatization of linear algebra around 1930. Each history is a particular interpretation, a convolution, not necessarily definitive, of a given problem. Here we are interested in the minimization problem, and the algebraic solution of the least-squares problem. This approach has been completely put aside by the statisticians, and in particular by Stigler in his book *History of Statistics* (1986). We briefly emphasize the errors of observations and the statistics during antiquity and the Middle Ages, and the importance of astronomy for the development of statistical methods. We discuss the genesis of the least-squares method in the 18th century. We present the work of Gauss, Laplace, Legendre and Bienaymé. In particuar, we comment on Gauss' proofs for the least-squares method. But this work concerns mainly the evolution of least squares and the theory of errors during the beginning of the 20th century by the minimization of the norms and also the approximation of periodic functions by Fourier series.

Aditi Gowri (University of Texas -- Austin)
(Saturday, 11 a.m.)

"Mathematical Revitalizations: A Model for the Social Historical Study of Foundations"

This paper offers the outline of a method for the historical study of mathematical foundations. Borrowing from Anthony Wallace's anthropological construct of the mazeway, I would like to present a model for analyzing the process in which an accepted set of justifications for the validity and plausibilty of mathematical knowledge (or foundations) can come under question, then undergo modification or reformulation. The key insight of my paper will be that foundational movements (like cultural revitalization movements more generally) are a response to some threat to the legitimacy of accepted knowledge and/or procedure.

Hardy Grant (York University)
(Sunday, 4 p.m.)

"M_{191} and the History of Factoring"

The history of attempts to factor large numbers can not (of course!) be told in half an hour; but it happens that the case of the Mersenne number M_{191} (i.e., $2^{191} - 1$) reflects several of the leading features, since this number's five prime factors were found by four different methods spanning more than two hundred years. I shall therefore structure an overview around the career of M_{191}, and then (if time allows) offer some thoughts on what a more comprehensive history of factoring might contain.

Peter Griffiths (independent scholar)
(Sunday, 5 p.m.)

"The Mathematics of the Projection Maps of Etzlaub (1511) and Mercator (1569), including comments from Edward Wright (1599)"

The projection maps of Etzlaub (1511) and Mercator (1569) are based on straight parallel lines of latitude and longitude. By applying some fairly simple trigonometry shown by Edward Wright (1599), Etzlaub and Mercator could have computed that the ratio of the radius and circumference of the Arctic Circle to the radius and circumference of the Equatorial Great Circle was cos 66.5°, that is, the cosine of the degree of latitude. On the globe, the great circles of longitude converge toward the poles. If, however, on a flat map these longitudes are shown not as great circles but as parallel straight lines, then the flat map will show an East-West distance exaggeration, particularly near the poles.

To complement this East-West distance exaggeration, both Etzlaub (1511) and Mercator (1569) introduced a similar distortion for North-South distances. This was achieved by progressively increasing the space between the parallels of latitude from the Equator to the North Pole. Mathematically, this progressive increase should take the form of Edward Wright's geometric conclusion that distances along small circles parallel to the Equatorial great circle were of the ratio to the great circle as the cosine of the degree of latitude is to 1. The reciprocal of this is the secant of the degree of latitude, which is the dustance between latitudes on the flat maps of Etzlaub (1511) and Mercator (1569).

Israel Kleiner (York University)
(Sunday, 11:30 a.m.)

"Proof: A Many-Splendored Thing"

In a recent article in the *Bulletin* of the AMS Jaffe and Quinn expressed concern about what they viewed as diminishing standards of proof in certain areas of mathematics, and suggested a framework for dealing with the issue which included attaching labels to "speculative and intuitive" work. The article has engendered a fascinating debate about the nature and function of proof in mathematics. I hope to show, using examples from the history of mathematics, that the current debate is a continuation of a 2000-year-old tradition.

Erwin Kreyszig (Carleton University)
(Saturday, 4 p.m.)

"Some Historical Roots of Modern Numerical Analysis"

Astronomy, geodesy, geometry, the theory of special functions, elementary number theory, and practical needs of economics caused the evolution of early numerical work, at first unrelated, and later more and more incorporated into systematic theories, in connection with table-making, algebra, integration, and the theory of differential equations. Some of these earlier ideas and methods have gained practical significance because of the advent of the computer, a development that began around forty years ago. in this talk we shall discuss early numerical work and historical roots of ideas and methods in modern numerical analysis, in the works of Newton, Euler, Lagrange and Gauss and their contemporaries.

Elaine Landry (University of Western Ontario)
(Saturday, 9:30 a.m.)

"Semantic Realism: Why Mathematicians Mean What They Say"

In this paper I argue that if we distinguish bewtween ontological realism (the claim that mathematical entities exist) and semantic realism (the claim that mathematical statements which talk about mathematial entities are meaningful), then we no longer have to choose between platonism and formalism. if we construe category theory as the language of mathematics, then a linguistic analysis of the content and structure of categories allows us to justify the inclusion of mathematical entities (objects and arrows) as legitimate objects of philosophical study. Specifically, such an analysis permits us to justify the claim that mathematical entites exist independently of us but, at the same time, depend on the structure (or form) of a given category and the content of a given theory.

Insofar as this analysis relies on a distinction between ontological and semantic realism, it further relies on an implicit distinction between mathematics as a descriptive science (the view that mathematics *is about* entities) and mathematics as a descriptive discourse (the view that mathematics *talks about* mathematical entities). It is this latter distinction, I argue, which gives rise to the tension between the mathematician *qua* mathematician and the mathematician *qua* philosopher. When the mathematician claims that a mathematical entity exists he intends to make a semantic claim. On the other hand, when the physical scientist claims that an entity exists he intends to make an ontological claim. Thus, when the philosopher comes to analyze "existence" claims, he must be careful to distinguish between these intentions. In conclusion, I argue that the tensions between formalism and platonism, indeed between mathematician and philosopher, arise because of an assumption that there is an analogy between mathematical talk and talk in the physical sciences.

Jacques Lefebvre (Université du Québec à Montréal)
(Sunday, 3p.m.)

"The importance and ambiguity of mathematics in the works of Robert Musil"

Robert Musil (1880-1942) was born in Austria and wrote in German. His initial professional training was in engineering. As a writer, he is noteworthy for the breadth of his mathematical knowledge and for the role mathematics plays in his works.

Learning mathematics is the main intellectual episode of his first novel and the "hero" of his major literary work is a mathematician. For Musil, mathematics is both useful for its applications and worthy of study for its own sake. What's more, the precision of mathematics and a trial-and-error attitude are seen as a tool for changing men's behaviour, values and concepts about human life.

But all is not without difficulty. The learning of mathematics is not without its mysteries. The foundations of mathematics are not sure and many of its applications are controversial. What's more, there is an obscure evil force at work in mathematics. There is also a counterbalancing tendency, just as fundamental as mathematics, that resists it. That tendency may be called fusion, love or emotion.

The duality is unresolved.

Michael Millar (University of Northern Iowa)
(Sunday, 4:30 p.m.)

"Newton's Segmental Neusis Construction of the Regukar Heptagon"

Some time around 1670 Isaac Newton prepared a manuscript entitled "Problems for Construing Aequations". In this work, Newton gives a number of geometric constructions, many of which are kinematically based. Of particular interest are problems 18, 19 and 20, in which he constructs -- without proof -- the third, fifth and seventh parts of a right angle. While the first two of these constructions represent only slight variations of those found in Euclid's *Elements*, we shall examine how Newton arrived at the third construction -- which of course leads to the regular heptagon -- using a "segmental", rather than an Archimedean "areal", neusis construction.

Michael Pool (University of Western Ontario)
(Saturday, 10 a.m.)

"Descartes' Early Philosophy of Mathematics and Perception"

This is a discussion of the standard interpretations of two significant elements of Descartes' "Regulae", viz., the "mathesis universalis" (universal mathematics) and the account of perception that occurs in Rule 12. Rule 12 has been interpreted as a robustly realist account of perception. I argue that the most plausible interpretation of Descartes' universal mathematics plus his appeal to mathematical knowledge as the standard by which knowlege is to be judged is in tension with this view. In light of this I argue for a non-standard, but more compatible, idealist interpretation of Rule 12. I indicate how this interpretation is relevant to the philosophy of mathematics by showing how it contains the roots not only of Kant's philosophy of mathematics but also of important elements of Hilbert's formalism.

Siegfried Thomeier (Memorial University)
(Saturday, 1:30 p.m.)

"C.F. Gauss: Mathematician, Scientist and Inventor"

It is well known that Gauss, in addition to being one of the greatest mathematicians of all time, also made significant basic contributions to several areas of science, especially in astronomy and physics (e.g., in optics, geodesy, geomagnetic research, and electromagnetism).

It is not so well known that, in the process, he also came up with various important practical contributions and inventions. We want to consider only a few such examples (and their interrelations with his mathematical and scientific main interests), including his invention and construction in 1833 -- in cooperation with his friend, the physicist Wilhelm Weber -- of an electromagnetic telegraph, which was operational in Göttingen for over a decade, and which predated the telegraph of the American Samuel Morse by several years.