Canadian Society

for History and Philosophy

of Mathematics

Société canadienne

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des mathématiques

PROGRAM/PROGRAMME

Sixteenth Annual Meeting
Seizième Congrès Annuel
May 31 - June 1   31 mai - 1 juin 1990
University of Victoria/ Université de Victoria
Victoria, British Columbia

Program/Programme

Thursday May 31/jeudi 31 mai

9:00 *Len Berggren,* President, CSHPM/SCHPM
Fran Abeles, Conference Organizer, Kean College, SCNJ
Welcome

JOINT SPECIAL SESSION: HISTORY AND PEDAGOGY/ SESSION SPÉCIALE COOPÉRATIVE : HISTOIRE ET PÉDAGOGIE 9:10-3:30 (Presider, Morning: *Victor Katz*)

9:10 *Victor Katz*, Organizer, Special Session
Introduction of guest speaker

9:15 *Judy Grabiner,* Guest Speaker
Was Newton's Calculus a Dead End?
Maclaurin's Place in British and Continental Mathematics

10:15 TEA & COFFEE/THÉ ET CAFÉ

10:40 *Israel Kleiner*
Themes in the Evolution of Number Systems

11:20 *Charles Jones*
The Beginnings of the New Math Movement:
The Ball State Program

11:50 LUNCH/DÉJEUNER
(COUNCIL MEETING/RÉUNION DU CONSEIL)
(Presider, Afternoon: *Israel Keiner*)

1:20 *Sam Kutler*
Why Study Ancient Mathematics?

1:55 *Erica Voolich*
A Multicultural and Historical Approach in the Elementary Classroom

2:30 *Victor Katz*
Non-Western Mathematics in the University Classroom

3:00 *V. Fred Rickey*
Old Calculus Problems Make for a Lively Course

3:30 TEA & COFFEE/THÉ ET CAFÉ

REGULAR SESSION/SESSION ORDINAIRE 3:55 - 5:15

3:55 *Irving Anellis*
The Roots of Mathematics and Mathematics Education in Russia in the Age of Peter the Great

4:35 *Erwin Kreyszig*
Plateau's Problem

Friday June 1/vendredi 1 juin

REGULAR SESSION/SESSION ORDINAIRE 8:30-5:25
(Presider, Morning: Craig Fraser)

8:30 *James Tattersall*
Nicholas Saunderson: The Blind Lucasian Professor

9:00 *Ubiratan D'Ambrosio*
The Life and Work of Joaquim Gomes de Sousa (1820-1863)
A Brazilian Analyst

9:30 *Craig Fraser*
The Mathematical Origins of Lagrange's
Theory of Planetary Perturbations

10:00 *Elizabeth Smith*
Scottish Contributions to the Introduction of Continental
Analysis in Britain, 1780-1815

10:30 TEA & COFFEE/THÉ ET CAFÉ

10:55 *Sharon Kunoff*
A Curious Counting/Summation Formula
from the Ancient Hindus

11:25 *Len Berggren*
Greek and Islamic Elements in Arabic Mathematics

11:55 LUNCH AND ANNUAL MEETING/DÉJEUNER ET RÉUNION ANNUELLE
(Presider, Afternoon: *Fran Abeles)*

1:30 *Glen Van Brummelen*
A Survey of Interpolation Methods from Harriet to Newton

2:00 *M.A. Malik*
Markoff's Theorem on the Derivative of Polynomials

2:30 *Liliane Beaulieu*
An Instance of Bourbaki's Decision-Making Process:
Modules in Linear Algebra (1941-1946)

3:00 *Jacqueline Brunning*
C.S. Peirce's Relative Product

3:30 TEA & COFFEE/THÉ ET CAFÉ

3:55 *Peter Griffiths*
Archimedes Computation of Pi, A Clarification

4:25 *Jonathan Seldin*
From Exhaustion to Modern Limit Theory

4:55 *A.K. Ray*
History of Space Mathematics

ABSTRACTS / RÉSUMÉS

Special Session on History and Pedagogy

J. Grabiner (Pitzer College). Was Newton's Calculus a Dead End? Maclaurin's Place in British and Continental Mathematics.

The standard picture of the history of the calculus follows its simultaneous invention by Newton and Leibniz with the death of the Newtonian tradition, while calculus in the Leibnizian school flourishes with the Bernoullis, Euler, d'Alembert, Lagrange, Laplace, Cauchy, Weierstrass, and on to modern analysis. Maclaurin's place, if he has one at all, is as a reactionary advocate of Newtonian fluxions and ancient geometry --or as the man who set x=0 in the Taylor series. In fact, Maclaurin, and through him the Newtonian calculus, played an important role in Continental analysis. This should not be surprising, because Maclaurin was a key figure in the vigorous Scottish university tradition, with its influential links to all of Continental scientific thought.

C. Jones (Ball State University). The Beginnings of the New Math Movement: the Ball State Program.

The Ball State Program was begun in the early 1950's and anticipated a number of the objections that became part of the "New Math" movement. The origins of the program and its relationship to the later National Science Foundation supported efforts are some of the questions considered.

V. Katz (University of the District of Columbia). Non-Western Mathematics in the University Classroom.

Often, the use of historical examples and motivations in university classes are from the western tradition and continue to foster the notion that mathematics was entirely a western invention. In today's classroom, with students whose ancestors came to North America from countries around the world, it is important also to use examples from non-Western cultures whenever possible. Some of these will be discussed, including material from China, India, Central America, and the Pacific Islands.

I. Kleiner (York University). Themes in the Evolution of Number Systems.

It has been said that all of mathematics can, in essence, be reduced to a study of number and shape. By exploring various themes in the evolution of number systems (from the natural through the complex numbers and beyond, to transfinite, p-adic, and hyperreal numbers) we can introduce algebraic, analytic, geometric set-theoretic, and number--theoretic ideas, as well as historical and philosophical issues, in a relatively concrete setting.

S. Kutler (St. John's College). Why Study Ancient Mathematics?

While I will mention several reasons, the chief among them will be that one can understand "modern" mathematics best when one contrasts it with how certain ancients practiced their craft, and most of all that one can understand modernity best when one views it through the eyes of Descartes, who is rejecting the ancients' practice of mathematics and advancing notions of his own, many of which have become "our" symbols and methods.

V. Rickey (Bowling Green State University). Old Calculus Problems Make for a Lively Course.

The hypothesis that our calculus course can be enlivened by the use of a variety of interesting problems from old calculus books and papers will be supported by a multitude of examples, including the Swineshead-Oresme series summation, Torricelli's trumpet, Leibniz' first differential, and many others.

E. Voolich (Wheelock College). Mathematics: A Multicultural and Historical Approach in the Elementary Classroom.

So often students finish elementary school believing that all of the mathematics that they have learned was handed down to Moses along with the Ten Commandments. In an effort to remedy this misconception, a multicultural and historical approach to teaching is used. Specific examples from lessons for middle school students will be shared.

ABSTRACTS / RÉSUMÉS

Contributed Papers

I.H. Anellis (Modern Logic). The Roots of Mathematics and Mathematics Education in Russia in the Age of Peter the Great.

A descriptive sketch is presented of mathematical knowledge in Russia during the 15th to 18th centuries as it developed in its cultural situation. Special consideration is given to the professional development of mathematics during the reign of Peter the Great (reigned 1682-1725), and to the origins and development of mathematical education in that period. It will be shown that mathematical knowledge in pre-Petrine Russia and in the years of Peter's reign was considerably more developed than is commonly believed. Particular emphasis will be placed on descriptions of the mathematical contents of the *Synodal 42 Geometry* a manuscript of the second half of the 17th century, and of the manuscript of Feofan Prokopovich's mathematical lectures of 1707-1708 at the Kiev-Mogila Academy. The latter will be compared with L.F. Magnitskii's better known book *Arifmetika* of 1703 which served as a textbook for much of the 18th century. An overview of elementary and collegiate education in mathematics including a discussion of the roles of Peter the Great, J.D. Bruce, Henry Farquharson, Feofan Prokopovich, and Stefan Lavorskii in formulating policies of mathematical education in Russia and the teaching of Magnitskii, Henry Farquharson, Prokopovich and A.D. Adodurov will be described as will be the history of the founding of the School of Mathematics and Navigation in Moscow (1701) and the origins and activities of the mathematics faculty of the Academy of Sciences and its students.

L. Beaulieu (Rosemont College). An Instance of Bourbaki's Decision-Making Process: Modules in Linear Algebra (1941-1946).

Bourbaki's Linear Algebra was first published in 1947, twelve years after its initial draft was drawn up by the team. In the course of those years, many changes occurred and many issues were discussed. This paper will follow one particular debate, from the first suggestion to introduce modules in linear algebra, in 1941 to the end of that discussion in 1946.

Two main questions were discussed by the members of Bourbaki in relation to the place of modules in linear algebra. The first question was whether linear algebra should be presented with the focus on vector spaces or whether it should be treated more generally, starting with modules. The second question arose over the need to do a whole section on modules in the second chapter of the algebra book whereas groups with operators were already introduced in its first chapter and thus could be referred to without much further development. This debate involved arguments of generality, aesthetics, economy and the use of concepts in related theories.

J.L. Berggren (Simon Fraser University). Greek and Islamic Elements in Arabic Mathematics.

It is beyond question that of the many elements constituting the body of material which Arabic mathematics had to work on in its early years, the material from the Greeks formed the largest part. (This assertion is true whether one restricts oneself to mathematics *per se* or includes metamathematical texts concerned with issues of permissible argumentation, the scope and organization of mathematics, etc.) Indeed recent research has only added to the list of Greek works known to Arabic mathematical writers and has underlined the importance of Greek philosophical assumptions to these writers. At the same time, other work has shown that there were other important elements which, because they developed out of the specific character of the Muslim faith, may be called Islamic and which shaped the acquisition of the Greek elements. These Muslim elements include religious injunctions and alternate traditions and attitudes towards knowledge and its function. In this paper which is in the nature of a survey of current work, we shall survey what has been learned recently about the process of transmission and how the acquired elements interacted with Muslim society to create a mathematics which can be understood neither apart from Greek mathematics nor as something which is only a continuation of Greek mathematics.

J. Brunning (University of Toronto). C.S. Peirce's Relative Product.

This paper will examine a particular historical and mathematical connection between the algebra of relations of C.S. Peirce and the Linear Associative Algebra of Benjamin Peirce. The claim is that the central definitional operation in the algebra of relations, relative product, was taken from the multiplicative operation of the Linear Associative Algebra of Benjamin Peirce. I show how many of the mathematical analogies in the early stages in C.S. Peirce's development of the algebra of relations are due to the influence of Benjamin Peirce's Linear Associative algebra. This is most explicit in C.S. Peirce's matrix formulation of relations in his 1882 paper, "Brief Description of the Algebra of Relations".

U. D'Ambrosio (Universidade Estadual De Campinas). The Life and Work of Joaquim Gomes de Sousa (1829-1863), A Brazilian Analyst.

Born in 1829 in the State of Maranhão, Northeast Brazil, from a well placed family, Joaquim Gomes de Sousa moved to Rio de Janeiro, the Capital of the Brazilian Empire, in 1844 and in 1848, nineteen years old, he submitted to the Military School (then the only institution of higher learning to grant advanced degrees) a thesis on Celestial Mechanics. Based on Laplace's results and the then recent discovery of Neptune (1846), Gomes de Sousa analysed pertubations of the motion equations due to the presence of another body. His works were presented to the Academie des Sciences de Paris (1855) and to the Royal Society of London (1856), where they received mention by G.G. Stokes. In 1855 he received a medical degree from the Université de Paris, at the age of 24. While still in Europe he was elected representative of his home state, Maranhão, to the National Assembly of Brazil and moved back to Rio de Janeiro. Since then until his untimely death at the age of 34, his activities were in the field of philosophy and the social sciences. His unfinished major work, in several volumes, was a general system of the world.

In this paper we look into the mathematical works of de Sousa popularly known in Brazil as "Sousinha", and into the cultural, social and political environment which allowed such a bright achievement by a young man from a peripherical state. We also try to understand why the promises of a bright mathematical career measured by European standards did not materialise, lasting no more then a few years, being put aside in favor of other intellectual and political interests. Our main conclusion leads to the need of a "mathematical, sustainable environment" for the institutionalisation of Mathematics rather than the appearance of isolated geniuses. It is also intriguing to ask how do these geniuses appear.

C. Fraser (University of Toronto). The Mathematical Origin of Lagrange's Theory of Planetary Perturbations.

Lagrange's contribution to the theory of planetary perturbations was the method of variation of parameters and the associated concept of a perturbing function. At the time he did this work, in the 1770s, he was also engaged in the study of ordinary and partial differential equations. This research concerned his very successful and original exploitation of the idea of varying arbitrary constants that appeared in their solution. Some sort of connection between his research in planetary astronomy and his theory of differential equations seems evident, and indeed has been suggested in the literature. However, there is at present no comparative historical study of the related content of these researches or of the precise circumstances of their production. The paper will begin to provide such an account.

P.L. Griffiths. Archimedes's Computation of Pi, A Clarification.

Archimedes's "On the Measurement of the Circle" is divided into two sections, the circumscribed section purporting to compute the upper limit for pi and the inscribed section purporting to compute the lower limit for pi.

Most of the fractions in "On the Measurement of the Circle" are either cotangents or cosecants of the angles 30^0, 15^0, $7\ 1/2^0$, $3\ 3/4^0$ and $1\ 7/8^0$.

The denominators of the fractions in the circumscribed section are all 153, whereas the denominators of the fractions in the inscribed section are mostly 780.

In the circumscribed section the fraction for $\sqrt{3}$ is $265/153 < \sqrt{3}$, whereas in the inscribed section the fraction is $1351/780 > \sqrt{3}$.

T.L. Heath complains that Archimedes gives no explnation as to how he arrived at these fractions.

Archimedes uses only one formula for both sections namely the cotangent half angle formula which can be variously expressed as

$\cot a/2 + \operatorname{cosec} a/2 = \cot a/4$

or $\cot a/2 + \sqrt{1 + \cot^2 a/2} = \cot a/4$

which is an iteration or recursion formula in that the cot a/4 result of one calculation can be used as the data of the next calculation. The proof of this formula makes use of a proof in Euclid's Elements book 6 proposition 3, namely that a line bisecting the angle of a triangle will cut the opposite side in the ratio that the other two sides bear to each other.

In theory only one instead of two applications of the cotangent half angle formula should give the upper and lower limits for pi. The reason for the two separate operations was the inaccuracy of Archimedes's fractions particularly the fractions for $\sqrt{3}$. Archimedes did not work in a decimal system.

E. Kreyszig (Carleton University). On Plateau's Problem.

This work is based in part on personal communications with the late Prof. Tibor Rado, who was the author's colleague at The Ohio State University. Differential-geometric concepts needed will be recalled.

Plateau's problem, named after the Belgian physicist J. Plateau, is the problem of determining a minimal surface bounded by a given Jordan curve in space. It was solved in 1930 by Rado and independently in 1931 by J. Douglas, one of the first two recipients of the Fields medal (1936).

In this talk we consider the history of the ideas on minimal surfaces that led to Rado's solution, under the influence of the calculus of variations (Euler, Lagrange, Hilbert, Haar), mapping problems (Gauss, Minding, Bonnet), complex analysis (in particular, Weierstrass's formulas of 1866), Lebesgue integration (since 1902) and "modern" theory of partial differential equations (beginning with Bernstein's thesis of 1904).

S. Kunoff (Long Island University). A Curious Counting/Summation Formula from the Ancient Hindus.

Most of the popular history of mathematics texts give the impression that contemporary mathematical throught stems directly from the resurgence of western creativity after the dark ages. However mathematical ideas continued to develop in India, the Middle East, and the Orient from their ancient beginnings. There is evidence that some of modern western mathematics travelled from these sources. Examples of Hindu mathematics appear in extant manuscripts from the eighth and ninth centuries. Many of the permutation and combination formulas attributed to Cardano, Tartaglia, and Pascal were known to the Hindus. Counting problems appear in their literature along with questions relating to the sums of the numbers permuted. These summation formulae which do not appear in modern texts will be considered here in their original (albeit translated) form and expressed using modern symbolism. The translation from the ancient verbal terminology to modern symbolic notation is an exercise that can be appreciated by a student of elementary combinatorics.

M.A. Malik (Concordia University). Markoff's Theorem on the Derivative of Polynomials.

In response to a question asked by the chemist Mendeleev, A.A. Markoff studied the problem:

Let P(x) be a polynomial of degree n and $|P(x)| <= 1$ on the interval [-1, 1]. Let also $a \in [-1, 1]$. Then how large can $|P'(a)|$ be?

In 1889, Markoff gave a partial solution of this problem in his work "On a problem of D.I. Mendeleev". In this talk, we describe the work of Markoff and also discuss the subsequent progress on this subject.

A.K. Ray (Fundamental Research Institute). History of Space Mathematics.

The present author intends to abbreviate the history of space mathematics in consequence of rationalization of scientific attitude towards development of natural sciences from Middle Age to the beginning of modern times when day dawned for Moon-flight leading explorations for deeper space. Starting with the nature of the bridge between sense perceptions and the concepts during pre-space era, Kepler (1571-1630) as human psyche identified imagery souls of the heavenly bodies while discovering the famous laws of planetary motions. These laws were to become one of the pillars of Newton's theory of gravitation whilist edging with Galileo's discovery of constant acceleration of freely falling bodies.

These discoveries are sudden enlightments called inspirations can not be produced by chance alone (Zb:Poincare) but mind thinks streneously either " by analogy or by habit" through "subjective or objective methods". The outcomes either are perhaps "the facts of jumping over intermediaries" an act of unconscious mental processes (as intuition or else) or represent remarkable "intermediary stage between the earlier symbolism and the modern quantitive-mathematical desriptions " of the discoveries (natural laws or else).

However, we are now embarking on a period of explorations and discoveries unmatched since 16th century. The world has witnessed an enormous development of science with wildest influence of technology.

At this stage, author cites the principle of "Noblesse oblige" thus freeing from obligations of having a complete and thorough knowledge in any fields of specialization of space science and ventures to outline the global implications of space right from the drawing board of the policy makers to the elegant but sensible mathematics in the realm of planetary motions.

J.P. Seldin (Concordia University). From Exhaustion to Modern Limit Theory.

The " method of exhaustion" used in ancient Greek geometry is closely related to the modern theory of limits and may be useful in teaching it.

Archimedes' proof of Proposition 1 in {\em Measurement of a Circle} is indirect. If the steps of the proof are written out in modern algebraic notation, if all of the inequalities are made explicit, and if the indirect proof is replaced by a direct one, the result is a proof about the limits of sequences in the $\epsilon-N$ style acceptable in modern analysis. Furthermore, the theorem itself can be visulalized easily if one considers a circle and inscribed regular polygons. Thus, this exercise might be useful for introducing that part of the theory of analysis.

A similar use can be made of Proposition 2 of Book XII of Euclid's {\em Elements}.

E. Smith (University of Toronto). Scottish Contributions to the Introduction of Continental Analysis in Britain, 1780-1815.

This paper examines the contributions of a small group of Scottish mathematicians, including John Playfair, James Ivory, and William Wallace, to the introduction of French analytical methods and notation into Britain. Their contributions are examined in the context of Scottish mathematics and philosophy and their setting within the Scottish universities.

J.J. Tattersall (Providence College). Nicholas Saunderson: The Blind Lucasian Professor.

Nicholas Saunderson (1682-1739) was the fourth Lucasian Professor of mathematics at Cambridge, which was a remarkable feat considering that he contracted small pox when he was a year old and lost his sight. It was said he was a lecturer who had not the use of his eyes, but taught others to use theirs. Saunderson was a Latin, Greek, and French scholar, an accomplished musician, the inventor of a palpable arithmetic (a precursor of the geoboard), and author of several mathematical works, all of which were published posthumously. Lord Chesterfield attended his lectures at Cambridge, Horace Walpole was for a brief time a student of his, and George II awarded Saunderson an honorary doctor of laws degree in 1728. The author spent five weeks last summer at Cambridge gathering information about Saunderson, reading both his works and lecture notes, and will report on his research.

G.V. Brummelen (Simon Fraser University). A Survey of Interpolation Methods from Harriet to Newton.

I will concentrate on the finite difference interpolation methods developed by Harriot, Briggs, Gregory, and Newton, with an emphasis on motives. The historical and mathematical connections between these four figures will be explored, as well as their theoretical awareness of the nature of polynomial interpolation. In particular, I will argue that that there is no need to assume any more than elementary mathematical skills in order to construct Briggs' complex interpolation scheme.