CANADIAN SOCIETY FOR HISTORY AND PHILOSOPHY OF MATHEMATICS

SOCIETE CANADIENNE D'HISTOIRE ET DE PHILOSOPHIE DES MATHEMATIQUES

Fifteenth Annual Meeting *Quinziéme Congrès Annuel*

May 29 30 mai

1989

Université Laval

Quebec City, Quebec

PROGRAMME

Pavillon Comtois (4206/ 2105)

SCHEDULE

(This year we are experimenting with the idea of some organized interaction with the Canadian Society for the History and Philosophy of Science. There are two sessions cross-listed for Monday, May 29 that would appear to be of interest to both groups. In addition, those arriving early on Sunday might like to attend the Stilman Drake Lecture: Francois Duchesneau; "Leibniz on the Principle of Continuity", Sunday, May 28 at 2:30 p.m. (Pavillon Pouliot Rm.2505).

MONDAY, MAY 29 **MORNING**

(PRESIDER: R. Eddy)

9:00	L. Berggren, President, CSHPM/SCHPM	Welcome

SPECIAL SESSION: THE HISTORY OF MATHEMATICS IN THE 18TH CENTURY

9:10	C. Fraser, Coordinator	Introduction of guest speaker, Professor J.L. Richards
*9:15	J.L. Richards (2105)	Rigor and Revolution
10:30	**COFFEE**	
10:50	G.R. Van Brummelen	Reasonable Expectations: 18th Century Solutions of the St. Petersburg Paradox
11:30	C. Fraser	Pure and Applied Mathematics in the Philosophies of D'Alembert and Kant
*11:30	A. Jones	Arithmetical Methods in Hipparchus's Astronomy. (Pavillon Pouliot 2505)

*(Cross-listing with the Canadian Society for the History and Philosophy of Science)

12:10 - 1:45 **LUNCH**

APRES MIDI

SPECIAL SESSION CONTINUED

(PRESIDER: M. Malik)

1:45	I. Anellis	The Roots of Mathematics and Mathematical Education in Russia in the Age of Peter the Great, 1700-2725.

CONTRIBUTED PAPERS

2:30	F. Abeles	Lewis Carroll's Method of Trees: Its origins in the work of C.S. Peirce
3:00	**COFFEE**	
3:30	J. Loase	$\sqrt{2}$: A Foundational Perspective
4:00	A. Shenitzer	The Story of Diophantine Geometry
5:15	**ANNUAL MEETING**	

TUESDAY, MAY 30 **MORNING**

(PRESIDER: F. Abeles)

8:30	P.L. Griffiths	The British Influence on Euler's Early Mathematical Discoveries.
9:00	U. D'Ambrosio	From Manoel de Azevedo Fortes to Joze Fernandes Pinto Alpoym: the emergence of Mathematics in Eighteenth Century Colonial Brazil

9:30	L. Berggren	What the Manuscripts Tell: The Scietific Work of Abū Sahl al-Kūhī
10:00	V. Katz	Why Mathematics
10:30	**COFFEE**	
11:00	S. Kunoff	Developments in the Solution of the Differential Equation $x^n y^{(n)}(x) - x^m y(x) = 0$
11:30	I. Kleiner	Rigor and Proof in Mathematics: An Historical Persepctive
	LUNCH	
12:45	**COUNCIL MEETING**	

APRES MIDI

(PRESIDER: I. Kleiner)

2:00	R. Herz-Fischler	The Shape of the Great Pyramid: A Mathematical Hysteria
2:30	A.K. Ray	Mathematics in Man's Universe
3:00	R. Thomas	Mathematics is a Sphere not a Klein Bottle
3:30	**COFFEE**	
4:00	A. Jones	Pappus's Notes to Euclid's Optics
4:30	J. Seldin	Reasoning in Elementary Mathematics

5:00	S. Kutler	New Theorems involving the Golden Section.
5:30	J. Bidwell	Babylonian Geometrical Algebra.

ABSTRACTS - RESUMES

Special Session: The History of Mathematics in the 18th Century

1. **Joan Richards (Brown University): Rigor and Revolution**

The paper examines the larger intellectual context of French mathematics in the late 18th century, investigating how interest in making Rigor the defining trait of mathematics developed.

2. **Glen R. Van Brummelen (Simon Fraser University): Reasonable Expectations: 18th-Century Solutions of the St. Petersburg Paradox.**

The paper examines how different solutions of the St. Petersburg paradox in the 18th century altered basic concepts of probability in order to reconcile commonsense views with an extended domain suggested by the mathematical theory.

3. **Craig G. Fraser (IHPST, University of Toronto): Pure and Applied Mathematics in Philosophies of D'Alembert and Kant.**

The paper presents a comparative study of D'Alembert and Kant's understanding of the relations between pure and applied mathematics. Works considered include D'Alembert's Traté de Dynamique (1743) and Kant's Kritik der reinen Vernunft (1781, 1787).

4. **Irving H. Anellis (Iowa State University): The Roots of Mathematics and Mathematical Education in Russia in the Age of Peter the Great.**

The paper presents a survey of mathematics in Russia 1400-1800, with emphasis on the professional development of the subject in the reign of Peter the Great, 1700-1725.

ABSTRACTS - RESUMES

(Contributed papers)

1. F. Abeles (Kean College of NJ): Lewis Carroll's method of Trees: Its origins in the work of C. S. Pierce.

In 1894, Lewis Carroll developed a mechanical method to test the validity of complicated multiliteral statements using a reductio ad absurdum arguement. The basic ideas are similar to those in Beth's "Semantic Tableaux" and their seeds appear in papers by Peirce and his students that appeared in 1883.

2. U. D'Ambrosio (Universidade Estadual de Campinas): From Manoel de Azevedo Fortes to Joze Fernandes Pinto Alpoym: the emergence of Mathematics in Eighteenth Century Colonial Brazil

Since its publication in Lisbon in 1720, *O Engenheiro Portugues*, de Manoel de Azevedo Fortes served as a basis for instruction in the Military Schools of Engineers and the reference work for artillerists in Portugal and the colonies. It is a well written work dealing with basic arithmetic and geometry and military plans, tactics and fortifications. It was complemented by the more theoretical work by the same author *Logica Racional, Geometrica & Analitica* in 1744. In the same year, Joze Fernades Pinto Alpoym, a Brazilian military engineer, publishes in Lisbon an important treatise *Exame de Artilheiro*, more theoretical and methodologically more modern, and four years later the same Alpoym wrote a more advanced treatise, *Exame de Bombeiro.* An analysis of these works and the scientific and political atmosphere in Portugal and Brazil period will be discussed. Reference will be made also to the first major attempt for independence in Brazil in 1789.

3. L. Berggren (Simon Fraser University): What the Manuscripts Tell: The scientific work of Abū Sahl al-Kūhī

The scientific legacy of the 10th-century scientist Abū Sahl al-Kūhī consists of thirty-two treatises ranging in length from a single page to over sixty pages, in type from research communications to pedagogical compositions and correspondence, and in subject matter from the astrolabe to mechanics and questions in pure geometry. Over the last year I have been studying this considerable body of material, preparatory to translating and publishing al-Kūhū's collected works. In this talk I will tell something of what the manuscripts studied so far reveal about al-Kūhī's works and scientific personality.

4. R. Herz-Fischler (Carleton University): The Shape of the Great Pyramid: A Mathemetical Hysteria

Various authors have tried to give a mathematical explanation for the shape of the Great Pyramid of Egypt: these theories have ranged from simple ones involving only integer ratios to those involving pi and the "golden number". This talk will discuss the genesis of the various theories and also the real method that the Egyptians are known to have used.

5. A. Jones (I.H.P.S.T., Victoria College, U of T): Pappus's Notes to Euclid's *Optics*

The 'theory of appearances' is a modern name for a branch of ancient optics that deals geometrically with what figures look like to an eye in three-dimensional space. Euclid's *Optics* sought to provide a theoretical foundation for this subject, but aside from this work we possess few ancient documents to indicate how far the theory of appearances progressed. A question of particular interest is, to what extent writers on optics treated problems of perspective, that is, not merely describing the appearance of a plane configuration seen from a point in space, but prescribing the configuration necessary to produce a desired appearance.

The present paper discusses notes written by Pappus of Alexandria (fl. A.D. 320) on Euclid's *Optics*. These notes are buried in the little-read Book 6 of Pappus's *Collection* which otherwise consists mostly of uninteresting problems in spherical astronomy. The passage on the *Optics* has consequently received less attention than it deserves. Pappus does not discuss the *Optics* as a whole, but singles out one theorem (of obvious astronomical relevance) about the appearance of a circle seen from a point outside its plane. The highlight of Pappus's excursus is a pair of propositions concerning the apparent centre of an obliquely viewed circle: first, Pappus determines the point in the circle that appears to be its centre for a given position of the eye; secondly, he constructs the locus of positions of the eye from which any given point in the circle will appear to be its centre. The historical significance of the passage is ambivalent while it confirms that geometers in antiquity formulated and solved fairly complex problems motivated by the theory of appearances, it gives little support to the supposition that such problems were systematically studied beyond the elementary level represented by Euclid's book.

6. V. Katz (University of the District of Columbia): Why Mathematics

A current trend in mathematics textbooks is the emphasis on applications. One often gets the impression that mathematics was developed because of its applications. For example, one sees written that Newton developed his calculus for its application to his study of gravity and celestial mechanics. A study of the historical record shows, however, that large portions of currently applicable mathematics was developed for its intrinsic intellectual interest, including Newton's calculus. Examples will be given from various periods.

7. I. Kleiner (York University): Rigor and Proof in Mathematics: An Historical Perspective

Standards of rigor have changed in mathematics, and not always from less rigor to more. The notion of proof is not absolute. Mathematicians' views of what constitutes an acceptable proof have evolved. I will sketch that evolution focusing on the following:

1. The Babylonians
2. Greek axiomatics
3. Symbolic notation
4. The calculus of Cauchy
5. The calculus of Weierstrass
6. The reemergence of the axiomatic method
7. Foundational issues
8. The age of the computer

8. S. Kunoff (Long Island University/C.W. Post Campus): Developments in the Solution of the Differential Equation $x^n y^{(n)}(x) - x^m y(x) = 0$

In the study of ordinary linear differential equations, series solutions which coverge rapidly for small values of the independent variable: are often impractical as a means of computing the solution when the variable becomes large, even when the convergence is assured. In the early 1800's, series, which formally satisfied some of these equations and which appeared to approximate solutions to these equations rather well for large values of the variable had been found. In many cases these series did not converge and were considered approximate solutions of the differential equation. Many problems arose when trying to relate these approximate solutions to the known actual solutions. The earliest work done in this field was in reference to specific differential equations. The equation $x^n y^{(n)}(x) - x^m y(x) = 0$ turns out to contain many of these early problems as special cases.

The development of symptotic solutions of the differential equation $x^n y^{(n)}(x) - x^m y(x) = 0$ and the related connection problems, are traced from G. Stokes' insight relating to the Airy equation, $y'' - xy = 0$, in 1857 through B.L.S. Braaksma's definitive paper on the subject in 1971. The seemingly intuitive approach of E.W. Barnes, writing in 1906 is compared with the Mellin Integral derivation used by later researchers. Results obtained by I. Bakken, J. Heading, H.L. Turrittin and H. Wyrwich are considered.

9. J. Loase (SUNY Westchester Community College): $\sqrt{2}$: A foundational perspective.

This researcher has studied the modern foundational perspectives underlying the development of the irrationals-including the philosophies of logicism, intuitionism, and formalism. The lecture will examine the status of $\sqrt{2}$ from these viewpoints. I will also present a proof that the Kth digit of an irrational cannot be generated from polynomial functions with rational coefficients. The talk will close with several research questions together with the recommendation that irrationals may be more appropriately thought of as a posteriori constructs.

10. A.K. Ray (Fundamental Research Institute, Canada): Mathematics in Man's universe.

'There is nothing over which a free man ponders less than death; his wisdom is to meditate not on death but on life.'
Man's universe is the miracle of creation with sparks of life infused in the tiniest little monocycle of a cell and launched on a lonely voyage to the Unknown.
Life is recreated into a larger unit not alone through aggregation of cells but through a marvellous quality of complex inter-relationship maintaining a perfect co-ordination of function.
Furthermore, Man's universe is also a part of the material world where money speaks the language.
The large but important and very much discussed question is: How can the events in Space and time which take place within the spatial boundary of Man's universe be accounted for through the language of mathematics alone?
The primafacie case under such circumstances is to seek for pattern recognition conforming some laws and regularities thus discovered when applied to the behaviour of systems which do not exhibit the structure on which those laws and regularities are based.
In the present discourse, the author ventures to embark on a synthesis of facts and theories albeit with seond-hand and incomplete knowledge in some fields of Bio-mathematics and Economics while endeavouring for their respective mathematical models.

11. J. Seldin (Concordia University): Reasoning in elementary mathematics.

Solving an equation $f(x) = g(x)$ to get a solution $x = a$ amounts to proving that

$$\text{if } f(x) = g(x), \text{ then } x = a$$

(Furthermore, checking that $f(a) = g(a)$ by subsitution amounts to proving the converse.) This means that one of the main differences between arithmetic and beginning algebra is that in algebra we use reasoning to determine the operations to perform to find a solution (while in arithmetic we know immediately what operations to perform). However, the reasoning used in mathematics is not identical to the reasoning used in daily life or in other fields (such as philosophy or law). For example, in mathematics we insist on proving results which are intuitively obvious (often from axioms which are much less intuitively obvious). This suggests that part of the problem some of our students have in elementary mathematics courses is getting used to the unusual kind of reasoining involved. In his book "The Evolution of the Euclidean Elements", Wilbur R. Knorr presents a theory of how the notion of proof may have evolved from proofs by diagrams (i.e., using diagrams and recognizing that they are sufficiently general to prove what is required) to proofs as sequences of statements (i.e., the transition involved the discovery of incommensurability and the development of a theory about it). Much of this material is easily understood by students in a beginning algebra course.

The purpose of this paper is to propose ways in which these ideas (and some additional relevant material on ancient Greek society) can be used to improve the teaching of elementary mathematics courses, beginning with a first course in algebra.

12. A. Shenitzer (York University): The Story of Diophantine Geometry.

Like projective geometry, diophantine geometry was discovered three times, first by Diophantus, then by Jacobi in Euler's work on elliptic integrals, and a third time by Poincaré. This talk tells the story.

13. R. Thomas (University of Manitoba): Mathematics is a sphere not a Klein bottle

This paper attempts to give reasons why Lakatos's complaint that the history of mathematics is blind and philosophy of mathematics empty on account of reciprocal ignorance is no longer true. I see the two subjects, but especially philosophy, as gaining by drawing closer to each other and closer to mathematics. Philosophy in particular seems to me to have made some progress (!) both of the sort that Lakatos was instrumental in encouraging and on account of recognizing that mathematics is a human activity with both public and private sides.

ADDENDUM

1. P.L. Griffiths: The British Influence on Euler's Early Mathematical Discoveries.

Even though there are very few direct connections with Britain in Euler's early life, it appears that the most important British influence on Euler arose from the mathematical books and periodicals (particularly the Philosophical Transactions) obtained from Britain by the libraries in St. Petersburg and Berlin.

The connection between the trinomial factors of $1^n \pm z^n$ discovered by de Moivre, and Euler's discussion of the relationship between summation series and product series, leading to Euler's evaulation of the summation of reciprocals raised to a given power are worthy of consideration. A terminal formula stated by de Moivre in the supplement to Miscellanea Analytica seems to give rise to Euler's constant. Euler's discussion of the relationship between summation series and product series seems to have given rise to hyperbolic sines and cosines. Euler states the infinite series for cotangents, but it seems that this had been previously given by Isaac Newton in the 1676 Epistola Prior. This particular series generated, with some adjustment, the Bernoulli numbers and the evaluation of the summation of reciprocals raised to a given power.

S. Kutler (St. John's College, Annapolis, MD): New Theorems involving the Golden Section.

The simplest and most beautiful exhibitions of a line cut in extreme and mean ratio should have been discovered in ancient Greece or its renaissance; however, they seem not to have been discovered until the 1980s by George Odom of Poughkeepsie, NY, and Samuel Kutler of Annapolis Maryland. They also lead to constructions for cutting a given line into a golden section. The three concentric circle theorem of Samuel Kutler has been generalized by means of a simple diophantine equation to have infinitely many solutions. The diophantine equation was studied by Diophantus himself and Fibonacci.