CANADIAN SOCIETY FOR THE HISTORY AND PHILOSOPHY OF MATHEMATICS/
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Learned Societies meetings, University of Toronto, 3-4 June 1974/
Réunions des Sociétés savantes, Université de Toronto, 3-4 juin 1974

PROGRAM/PROGRAMME

3 June / 3 juin   123 Lash Miller

1:00-3:00   Charles V. Jones (Math/York), *Chairman*

Tyrone Lai (Phil/Memorial): "Did Newton Renounce Infinitesimals?"

De Morgan, in an aricle published in 1852, advances the thesis that Newton "renounces and abjures" the infinitely small quantity in 1704. My paper will establish that Newton did not; that infinitesimals formed in fact part of the foundation of his method of fluxions; and that, in addition, they are elements in his general ontology. Conceptual problems regarding infinitesimals were shelved by mathematicians at Newton's time for various reasons; Newton relegated these problems to a secondary position on metaphysical grounds.

Stephen Rogoczei (Math/Toronto): "Was Pure Mathematics Really Discovered by George Boole in 1854?"

An inquiry into the historical origins of the split between pure and applied mathematics can hardly ignore Russell's 1901 remark, that "Pure mathematics was discovered by Boole, in a work which he called the Laws of Thought (1854)". In the present paper various descriptions of pure mathematics are examined in addition to what Russell considers to be pure mathematics. The claims of Russell's 1901 paper regarding Boole's achievements are checked against the actual writings of Boole published in 1847 and 1854. Certain discrepancies are noted. In conclusion, the "discovery theory" of the origins of pure mathematics is contrasted with other explanations, especially as they relate to the present discontent and soul searching within the mathematical communities of both Canada and other countries.

Byron E. Wall (IHPST/Toronto): "The Calculus of Feeling: F. Y. Edgeworth's Quantification of Utilitarianism".

Jeremy Bentham's utilitarian principles were guiding lights to nineteenth century thinkers who sought to revitalize the moral sciences with a single unifying key. In 1881 F. Y. Edgeworth attempted a synthesis with models composed of continuous functions. He uses Gustave Fechner's just perceivable increment as a unit of feeling, and the calculus of variations to obtain inequality relations where precise measurement was impractical. His treatment of economics concentrates on a study of contract. To resolve indeterminate contract, he introduces the utilitarian calculus. In a "Euclidean" axiomatic method he derives theorems concerning the best distribution of means, labour, and birth rate so as to maximize the triple integral over happiness, individuals, and time.

V. Linis (Math/Ottawa): "Kant and Axiomatizations of Arithmetic"

Apart from the frequently refuted and ridiculed proposition that arithmetic is "a pure science of time", Kant's writings contain important analysis of the basic notions of arithmetic. It is a purpose of this paper to present in a concise manner those notions which were relevant to the subsequent developments in the axiomatization of arithmetic in the 19th century.

Gregory H. Moore (IHPST/Toronto): "Can Every Set Be Well-Ordered? A Turn-of-the-Century Controversy Leads to Axiomatization"

I consider the problem of well-ordering from Georg Cantor's original claim in 1883 that any set can be well-ordered to Ernst Zermelo's axiomatization of set theory in 1908. Many mathematicians rejected Cantor's proof, which required an infinite number of dependent choices. In 1904 Zermelo formulated the Axiom of Choice in order to provide an alternative proof. Through public letters the French mathematicians Baire, Borel, and Lebesgue opposed Zermelo's proof while Hadamard alone defended it. The three opponents emphasized that the proof was in no sense constructive and that the function used to well-order a given set was not well-defined. In 1906 Poincaré accepted the Axiom of Choice as a legitimate, synthetic a priori postulate but rejected Zermelo's proof for its use of impredicative procedures. Russell, Peano, and Brouwer objected to the Axiom for a variety of reasons.

In 1908 Zermelo replied to his critics. He stressed the intuitive evidence for the Axiom of Choice as well as its utility in proving many results in the theory of cardinal numbers and some in analysis. To strengthen his position, Zermelo stated the first axiomatization of set theory--explicitly including the Axiom of Choice.
In conclusion I discuss the evidence which my topic provides on the conjecture that axiomatization has arisen as a way of dealing with contradictory mathematical results or differing philosophical views.

Peter K. Schotch (Phil/Dalhousie): "A Modality and Contingency Problem in the Formalization of Science"

This paper deals with an issue connected with the program of the formalization of natural science. Many physicists and mathematicians, for example, are now engaged in research directed towards developing mechanics as an axiomatic discipline. The usual approach to the laws of mechanics is to regard them as not being necessarily true (but rather contingently true). Certain implications of this program are drawn out regarding the impact of this program on the contemporary formal logic of modality. In particular it is shown that the above program forces one to reassess the position of the modal system S5 usually taken to be one of the most intuitively acceptable.

7:30 General Organizational Meeting 158 Lash Miller

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4 June / 4 juin 161 Lash Miller

9:00-11:00 THE ROLE OF MATHEMATICS IN THE HISTORY OF PHYSICAL SCIENCE (Joint session with the Canadian Society for the History and Philosophy of Science)

Kenneth O. May (Mathematics/IHPST, Toronto) *Chairman*

John L. Berggren (Mathematics, Simon Fraser): "Archimedes' Contributions to the Development of Statics"

The purpose of this paper is to assess Archimedes' role in giving a mathematical formulation to the science of statics. The existence of Archimedes' work and its derivatives was of great importance for the development of a science of mechanics both in the Middle Ages and in the Renaissance. But, in addition, his work on statics played a fundamental part in the rest of his mathematical endeavours - both as a source of problems and in providing a method of discovering difficult mathematical theorems. Certainly from a logical standpoint (and internal evidence suggests from a chronological standpoint as well) all of his subsequent work rests, either directly (for its proof) or indirectly (for its discovery), on some of the theorems proved in Book I of "On the Equilibrium of Plane Figures." This paper therefore concentrates on that work. Previous students of Archimedes' work (Heiberg, Heath, Dijksterhuis, Mugler, et al.) have accepted the propositions of this book as emmanating from Archimedes - though some have remarked it seems to be a fragment of a larger work. We shall argue that, on the contrary, at least five and perhaps seven of the fifteen propositions in the text are not Archimedean. The extremely loose logical structure of the work, the trivial nature of some of its propositions, and an error in the proof of a major proposition all argue against it being an authentic piece of Archimedes' work. We shall compare it in some detail with the work "On the Sphere and the Cylinder" (Book I) in order to see more clearly the differences between the work under consideration and a real piece of Archimedes' mathematics. It will appear from the discussion that what we have is rather an instructional text in mathematical statics which derives only in part (much the best part, however) from Archimedes' work on the subject.

Stillman Drake (IHPST, Toronto): "Continuity and Discreteness in Early Theories of Free Fall"

Aristotle in his Physics defined "continuous", "contiguous", and "successive" quite clearly. The modern concept of the continuum hinges on Euclid Book V, but Euclid did not there use the word "continuous". Arabic alterations of Book V introduced the idea of continued proportionality, used by Medieval scholars in mathematicizing continua. Until the mid-16th century, the theory of proportion was essentially based on Euclid Books VII and VIII, and was arithmetical in character. A quantum aspect was thus introduced into mathematical physics, particularly with regard to speeds in free fall. This has been neglected by historians of impetus theory and its account of acceleration.

New Latin and Italian translations of Euclid in the 16th century restored the Eudoxian theory of Book V, making possible the rigorous treatment of ratios of continuous magnitudes. This was followed by Galileo's discovery of the law of free fall and its derivation by the use of one-to-one correspondence between infinite aggregates. The beginnings of the calculus

at the hands of Cavalieri and his method of indivisibles ensued.
Conservative physicists of the 17th century offered a means of reconciling Galileo's analysis with the quantum picture of the Medieval mathematical physicists. Although their programme was forgotten, it enables us to reconstruct Medieval physical thought on the one hand, and on the other to understand the disturbance caused in the early 20th century by the reappearance of quantum conceptions in motion.

H. S. M. Coxeter (Mathematics/Toronto): "The Space-Time Continuum"

Among the pure mathematicians who contributed to the rise of physical science, I would mention DESARGUES and PONCELET, who created projective space by adding ideal elements to Euclidean space; also CAYLEY and KLEIN, who used a polarity to equip this projective space with a non-Euclidean metric; RIEMANN and SCHÄFLI, who first understood that an n-dimensional continuum can be of finite extent without having a boundary (not only when $n = 1$ or 2 but also for greater values); STUDY, who boldly stepped outside KLEIN's absolute quadric to discover the exterior-hyperbolic space which DU VAL (seventeen years later) identified with DE SITTER's world, thus providing a convincing explanation for the observed departure of the most remote objects in the universe. I would mention also CLIFFORD, one of whose "geometric algebras" has 32 unitis which are now seen to be isomorphic to DIRAC's 32 matrices. Most particularly I would mention MINKOWSKI, who enriched affine space by inserting a real isotropic cone at every point, and invented the world line (which was so fruitfully developed by ROBB and SYNGE). MINKOWSKI might well have anticipated the theory of relativity if his brilliant career had not been cut short by untreated appendicitis.

ABSTRACTS of PAPERS BEING PRESENTED at the CANADIAN MATHEMATICAL CONGRESS, UNIVERSITE LAVAL, June 7 and 8, 1974.

Wei-Ching Chang (Math/Toronto): "Variants of the Chi-Square Test: Thiele's Conditional Binomial Test and Bowley's Chi-Dash-Square"

As a foe of the Bayesian method of inference, the Danish astronomer T. N. Thiele (1872) proposed a direct method of the conditional binomial test to judge the goodness of fit of a mortality table graduation. His techniques of linearlization and orthogonal transformations anticipated later works of Neyman (1949) and Irwin (1949). It turns out that this test is a special case of K. Pearson's (1900) chi-square test.
Pearson's test was employed in the English economic statistician A. L. Bowley's (1926) famous work on the sampling theory. By postulating the continuity of the *a priori* distribution, Bowley obtained the chi-dash-square test--the Bayesian counterpart of the chi-square test. His work, therefore, foreshadowed those of Neyman (1929), Jeffreys (1938) and Lindley (1965).

Kenneth O. May (IHPST, Math/Toronto): "Logical Fetishism and Mathematical Policy"

In a number of respects the dominant ideology of mathematicians is in conflict with reality, especially with the historical development of mathematics and with the requirements of a mathematical policy that would support a healthy growth of the discipline. This paper argues that the root difficulty is logical fetishism--the "blind reverence" for logic and the gross exaggeration of the significance of its use in mathematics.

Gregory H. Moore (IHPST, Toronto): "An Historical Perspective on the Axiomatization of Set Theory"

What gave rise to E. Zermelo's axiomatization of set theory in 1908? The standard response is that the set-theoretic paradoxes--such as Russell's paradox and the Burali-Forti paradox--were responsible. However, a careful analysis of Zermelo's papers suggests an alternative explanation.
Using a new postulate, which he later named the Axiom of Choice, Zermelo proved in 1904 that any set can be well-ordered. Quickly his proof provoked an intense controversy involving mathematicians in France (R. Baire, E. Borel, J. Hadamard, H. Lebesgue, H. Poincaré), Germany (F. Berstein, A Schoenflies), England (P. Jourdain, B. Russell), and Italy (G. Peano).

In 1908 Zermelo published two lengthy and closely related papers on set theory. In the first he gave a second proof of the well-ordering theorem, again by means of the Axiom of Choice. But he devoted most of that paper to an energetic refutation of those who had attacked either his earlier proof or the Axiom. He carefully phrased the new proof to fit into the axiomatization appearing in his second paper.
The evidence indicates that the motivation for Zermelo's axiomatization was twofold: (1) to provide a secure base for his Axiom of Choice by embedding it in an axiom system for set thoery, and (2) to provide an adequate foundation for both Cantor's set theory and all of mathematics. In these two papers Zermelo devoted little attention to the paradoxes. When he did use them, it was primarily to fault his opponents such as Peano. For Zermelo the paradoxes were only an obstacle to remove with as little fuss as possible--in contrast to Russell for whom they were a preoccupation.

Stephen Regoczei (Math/Toronto): "The Impact of Non-standard Analysis On Studying The History Of the Calculus"

As Abraham Robinson pointed out in his book, Non-standard Analysis (1966), the creation of a coherent system of analysis using infinitesimals necessitates a reexamination of the entire history of the calculus. Although non-standard analysis can be a valuable tool in carrying out this task, it alone is not sufficient to cope with the variety of approaches we find in 18th and 19th century primary sources. To better appreciate the complex thought patterns in these sources, one needs to draw several distinctions. One must distinguish concepts such as the infinitesimal, the indivisible, and the vanishing quantity, as well as the different versions of the continuum, such as Aristotelian, Galilean, Leibnizian, Weierstrassian, and Robinsonian. Examples from the works of l'Hôpital, Lazare Carnot, Cauchy, and Paul du Bois-Reymond are used to illustrate these distinctions.