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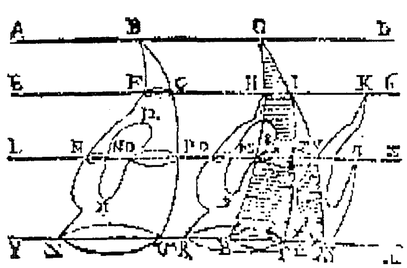
WHAT'S INSIDE



Canadian Society for History and Philosophy of Mathematics

Société canadienne d'histoire et de philosophie des mathématiques

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ABOUT THE SOCIETY

Founded in 1974, the Canadian Society for the History and Philosophy of Mathematics/ Société canadienne d'histoire et de philosophie des mathématiques (CSHPM/SCHPM) promotes research and teaching in the history and philosophy of mathematics. Officers of the Society are:

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The Society's Web page, designed and maintained by Glen Van Brummelen, is at www.kingsu.ab.ca/~glen/cshpm/home.htm

New members are most cordially welcome; please contact the Secretary-Treasurer.

An e-mail directory of CSHPM/SCHPM members is available on the Society's Web page at www.kingsu.ab.ca/~glen/cshpm/home.htm. For the current version please send to Glen, at the address above, an e-mail message with the subject heading "SEND CSHPM DIRECTORY".

Conference on Medieval Mathematics

A conference on medieval mathematics will be held on September 18-19 at Kellogg College, Oxford. Mathematics encompassed a diversity of activities in many medieval cultures and this weekend will aim to reintegrate the textual tradition of the medieval mathematical sciences with their instrumental and practical dimension. As a great deal of the world's mathematical heritage has found its way to the museums, libraries, and colleges of Oxford, it is an excellent location in which to study the whole context of medieval mathematical activity. Lectures on the manuscripts, instruments, and mathematicians of the period will be supplemented by an afternoon of tours and workshops at Merton College.

Speakers will include Silke Ackermann (British Museum), Sarah Bendall (Merton College), Karine Chemla (Paris), David Fowler (Warwick), Stephen Johnston (History of Science Museum), George Molland (Aberdeen), John North (Groningen) and Emilie Savage-Smith (Oxford). The conference is arranged by our sister society the BSHM and by the Department for Continuing Education, University of Oxford. The organizers are Raymond Flood, Department of Continuing Education, Oxford, raymond.flood@conted.ox.ac.uk, and Eleanor Robson, Oriental Institute, Oxford, eleanor.robson@wolfson.ox.ac.uk.

Something for Everyone: Exercises in the Philosophy of Mathematics

Madeline Muntersbjorn

Rewards and Challenges: Many agree that mathematical knowledge seems to transcend temporal and cultural boundaries. But why mathematics has this property, and how we ought to describe it, are open questions. An obvious reward of teaching philosophy of mathematics is an increased appreciation of the beauty, scope and power of mathematics. A subtle reward is renewed appreciation of the fact that, while mathematics itself generates unrivaled consensus, here too the *reason* is much disputed. The diverse academic backgrounds of our students pose the biggest challenge to philosophy-of-mathematics teachers. In theory, one would like to expect all students to have familiarity with the calculus, first-order logic, and set theory, plus prior exposure to the writings of Plato, Kant, and Frege. In practice, these expectations are unrealistic for most educators. The challenge is to set exercises that do not require extensive background in either mathematics or philosophy but call attention to pivotal features of both disciplines. Good assignments are both accessible to beginning undergraduates and challenging to advanced graduate students. Below are two exercises satisfying these parameters. The first is a weekly assignment; the second is suitable for a major paper or final project.

Word-Journal Technique: The word-journal technique is a note-taking strategy (1). Students keep a journal of keywords, definitions, and brief accounts of the relevance of these things to the course. There are three basic steps to constructing a word-journal entry. First: students consult specialized encyclopedias and dictionaries that are, in themselves, giant word journals. They are asked to compare and contrast various entries for the concept they are defining. Second: students relate the concept to the content of the course. Since the consulted reference works are written as general guides, students need to connect the concept explicitly to the philosophy of mathematics. Third: students identify the role the concept plays in discussion questions proposed throughout the course. Entries are judged on their style, clarity, coherence, conformity with canonical usage, and originality. I usually set restrictive limits on the size of each entry, about 300-600 words, and grade on a five point scale. However, class size and temperament should be taken into account when assigning a metric to this exercise.

This note-taking strategy is a good warm-up exercise in philosophical thinking that prepares students to write coherent and lucid philosophy papers. Restricting the length of the entries encourages the virtue of revision and discourages the vice of rambling. The technical vocabulary philosophers employ — especially the various "isms" — reflect systems of classification scholars have developed to make distinctions between philosophical positions. Keeping a word journal helps students make sense of pronouncements such as "Mill is an empiricist but Frege is a logicist". However, the difficulty of the word-journal exercise reveals the limits of such statements and teaches students to be wary of trying to force complex insights into rigid conceptual categories.

The table below presents two lists of words. The A-list contains technical vocabulary rarely employed outside philosophical circles and readily found in philosophical reference works. The B-list contains general vocabulary which acquires specific significance in the philosophy of mathematics. Preliminary definitions of these words may be found in ordinary dictionaries and students are encouraged to consult several. How many and how often word-journal entries are assigned depends on many factors, especially class size. For a class of 10-15 undergraduates, I assign two or three keywords at a time, one from the A-list and one or two from the B- list, every other week or so.

A:	B:
a priori/a posteriori	axiom certainty
analytic/synthetic	completeness
apodeictic	consistency
constructivism	construction
deduction/induction	content definition
empiricism	demonstration
epistemology	diagram experience
formalism	fallibility form
intuitionism	heuristic imagination
logicism	independence
modality	intuition knowledge
naturalism	necessity object
ontology	proof reason
Platonism	representation
realism	symbol
structuralism	understanding

Students often begin philosophy-of-mathematics classes confident they already know the meanings of B-list words. The A-list represents the diction they expect to learn. I ask students to define "easy" B-list words such as "diagram", "proof", and "object" off the top of their heads as in-class writing exercises during the first two weeks of the semester. This exercise reveals the naiveté of their initial confidence and encourages the habit of expressing their views on paper from day one. Students are also asked to consider the difference between related terms such as "representation" and "symbol", "certainty" and "necessity", "imagination" and "reason", etc. One of the lessons the word-journal exercise teaches is that B-list words are more complex and difficult to define than A-list words. Throughout the course I remind students that determining the best meanings which may be given to B-list words is an ongoing project pursued by both mathematicians and philosophers.

By pairing keywords and assigned readings carefully, we prime students to read primary sources. For example, what does Plato mean by "knowledge" and "object"? What does Kant mean by "intuition" and "construction"? What does Frege mean by "form" and "content"? I pick the best entries submitted on each keyword and distribute copies to the class as handouts, thereby teaching students the value of comparing notes. Beginners, often intimidated by the reputation philosophy of mathematics has for being abstruse, undertake this elementary exercise without fear. Advanced students quickly learn that defining "content", "experience", and "realism" in 600 words or less is difficult to do well. All students acquire insight into the concepts people invoke when engaging the discipline's central questions: what are mathematical objects? And, what is mathematical knowledge?

Dialog Project: The dialog project addresses another central question, what is mathematical progress? On the one hand, only a philosopher could seriously doubt whether mathematics is more clear, comprehensive and powerful than ever before. On the other hand, even those who regard the progressive growth of mathematics as indubitable have a difficult time explaining this growth, in part because of an inherent circularity: you cannot know what mathematical progress is until you have studied the history of math but you cannot study the history of math until you know what mathematical progress is. The dialog project is designed to help students confront this circle by asking them to follow the example of Imre Lakatos.

Lakatos argued that there is no such thing as history free from bias: all historical accounts are reconstructions of the past interpreted from the perspective of the present (2). His rhetorical strategy is to articulate philosophical theses about the nature of mathematical progress in fictionalized narratives. Lakatos presents a model of mathematical progress via a dialog between imaginary students. This dialog recreates the history of Euler's polyhedra conjecture; the details of the "actual" history are carefully presented in the footnotes accompanying the dialog. Participants paraphrase significant points made by past mathematicians. But they do more than spout lines from the history of mathematics. Their discussion as a whole is a compelling argument for Lakatos's account of scientific progress, namely, his model of conjectures and refutations.

The dialog project asks students to write an eight-to-ten-page dialog about an episode from the history of mathematics. After weeks of rigid word-journal writing, students embrace the potential for creativity in this project. Each student researches a particular episode and presents his or her case study to the class. The list below presents six sample episodes in chronological order. Each episode is based on a primary text readily available in an English translation and has been discussed by more than one author in the secondary literature. Episode summaries should be brief yet informative. Students must figure out for themselves whether and why their episodes exemplify mathematical progress.

Episode 1: Hippocrates of Chios "squared the lune" by showing that a given crescent-shaped area is equal to a determinate square area. Ivor Thomas's *Selections Illustrating the History of Greek Mathematics* presents a translation of Simplicius' version of Eudemus' account of Hippocrates' solution (Harvard, 1939).

Episode 2: George Berkeley criticized the calculus in *The Analyst* in 1734. Among other things, he claimed infinitesimals lead to the kinds of logical and metaphysical pitfalls rational thinkers try to avoid. In Volume III of his collected works edited by A. C. Fraser (Oxford, 1901) and elsewhere.

Episode 3: Initially, the Königsburg bridge problem was a local brain teaser: prove that you can not walk exactly once across each of the seven bridges connecting the banks of the river and the island downtown. D. J. Struik presents part of Leonhard Euler's general solution to this problem in *A Source Book in Mathematics 1200-1800* (Harvard, 1969).

Episode 4: Augustin-Louis Cauchy's limit formulations redressed ambiguities in the calculus. Initially presented in his (1821) *Cours d'analyse de l'École Royale Polytechnique*(1821) and *Résumé des leçons sur le calcul infinitésimal* (1823); extracts from these books may be found in Garrett Birkhoff's *A Source Book in Classical Analysis* (Harvard, 1973).

Episode 5: George Boole wrote *Investigation of the Laws of Thought* (1854), the first book devoted exclusively to what is now known as symbolic logic. Boole departed from tradition by employing algebraic symbols to represent set membership and logical operations (union, intersection, identity, etc.). Originally written in English and republished by Dover in 1951.

Episode 6: Giuseppe Peano's *The Principles of Arithmetic, Presented by a New Method* (1889) presented a set of axioms and notation to represent the foundations of arithmetic via symbolic logic. Extracts from this book may be found in Jean van Heijenoort's *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931* (Harvard, 1967).

Students can select episodes best suited to their respective academic backgrounds. Episodes 1 and 3, for example, do not require more than pre-college mathematics; episodes 2 and 4 require some familiarity with the calculus. Episodes 5 and 6 are good for students who have studied symbolic logic or computer programming. Of course, this list is but an indication of the projects possible. In one class, an English major wrote an insightful dialog about the poet Omar Khayyam's treatment of cubic equations. His lack of an extensive background in mathematics did not prevent him from seeing the historiographic challenges posed by Khayyam's "incipient algebra." Another student, a triple major in mathematics, physics, and philosophy, wrote a dialog on Erwin Schrödinger's 1926 proposal that his wave-mechanical formalism was equivalent to the quantum-mechanical models of the same phenomena as developed by Werner Heisenberg and others. His dialog was an insightful examination of the complex interplay between mathematical equivalence, experimental verifiability, and conceptual identity. (See **Exercises** p. 8)

Bernard Hodgson Wins Award

Long-time CSHPM member Bernard Hodgson of Laval University was honoured this past December by receipt of the fourth Adrien Pouliot Award, presented by the Canadian Mathematical Society. The citation read:

"Bernard Hodgson's contributions to mathematics education define a very high standard of service and achievement. The span of his activities is remarkable from a variety of perspectives. His involvement ranges through all tiers, from elementary to post-secondary mathematics education. It is marked by a long record of direct engagement with teachers and teacher candidates, by publications and presentations at all levels, and by participation and leadership in local, provincial, national and international mathematics education organizations. His contributions have been significant, sustained, and influential."

"Bernard Hodgson's remarkable energy, generosity, and wise counsel have benefitted numerous groups ... He has worked assiduously and effectively to build bridges between Canadian mathematics educators from different backgrounds, and between Canadian mathematics educators and their colleagues in other countries. The CMS is pleased to grant him the 1998 Adrien Pouliot Award."

Congratulations, Bernard!

> "Father Pirrone thought what a mess the world must seem to one who knew neither mathematics nor theology." — Giuseppe di Lampedusa, *The Leopard*

A message from the President

Jim Tattersall

It was heartwarming to see such a strong historical presence at the AMS-MAA Joint Meeting in San Antonio this past January, where about 3700 mathematicians gathered. Among the events of interest was a Joint Special Session on History of Mathematics organized by Karen Parshall and Victor Katz, the MAA Special Presentation on the Use of History in the Teaching of Mathematics organized by Fred Rickey and Florence Fasanelli, and an AMS Session on History of Mathematics. These sessions accounted for thirty presentations on the history and philosophy of mathematics. Over 450 attended, and over 100 had to be turned away from, the MAA dramatic presentation on the mathematics of Lewis Carroll, organized by Robin Wilson of the Open University. Participants in the performance included John Fauvel, Florence Fasanelli, Fred Rickey, Jennifer Beineke, and June Barrow-Green. The history of mathematics is truly alive and well at the AMA-MAA Joint Meetings.

The next such meeting, scheduled for January 19-22, 2000 in Washington, D.C., will feature sessions on history and philosophy of mathematics on all four days. In particular, Karen Parshall and David Zitarelli are organizing an AMS-MAA Joint Special Session on the History of Mathematics and Fred Rickey, Florence Fasanelli, and Victor Katz are organizing another MAA contributed-paper session on the Use of History in the Teaching of Mathematics.

There are a number of history of mathematics activities planned for the next six months that members should be aware of. As part of its "Emphasis Year in the History of Mathematics," the University of Virginia will be holding a 3-day symposium, entitled "Mathematics Unbound: The Evolution of an International Mathematical Community, 1800-1945", May 27-29. According to the organizers, Karen Parshall and Adrian Rice, this symposium aims to shed light on the historical processes and interactions involved in the development of what has become an international community of mathematicians. For more information contact Adrian Rice at ar6n@virginia.edu.

I hope that many of you will be able to make it to Providence, Rhode Island this summer to participate in Mathfest 1999 (July 31-August 2). Among the featured speakers are Jeremy Gray, who will give an MAA Invited Address on "100 Years of Hilbert's Problems", and Fred Rickey, who will give the Pi Mu Epsilon/J. Sutherland Frame Lecture entitled "The Creation of the Calculus: Who, What, When, Where, Why". The many activities at the Mathfest include general and graduate student contributed-paper sessions, a Newport Excursion, and a Rhode Island Clambake.

If you can't join us in July in Providence plan to come to the Ocean State in the fall when the foliage is at its peak. Daniel Otero of Xavier University and Edward Sandifer of Western Connecticut State College are organizing a special session on the history of mathematics at the American Mathematical Society sectional meeting at Providence College, October 2-3.

With regard to Y2K, the International Mathematical Union has declared the year 2000 to be World Mathematical year (WMY 2000). WMY 2000 is sponsored by UNESCO and many other scientific organizations. The Canadian Mathematical Society will sponsor special joint sessions of related academic societies at CMS meetings in June 2000 at McMaster University in Hamilton, Ontario and again in December 2000 at the University of British Columbia in Vancouver. Richard Kane, President of the Canadian Mathematical Society has extended an invitation to the CSHPM/SCHPM to schedule our annual meeting in 2000 with the CMS in Hamilton. The Executive Council has accepted this invitation. Other organizations meeting with the CMS at McMaster June 10-13, 2000 as part of the WMY 2000 celebration include the Canadian Applied and Industrial Mathematics Society (CAIMS) and the Canadian Symposium on Fluid Dynamics (CSFD). Among the activities of interest to our members will be a joint special session on history and philosophy of mathematics, organized by Tom Archibald, and several contributed-paper sessions. I realize that this will mean not meeting with the Congress of Social Sciences and Humanities (the "Learneds") for two consecutive years. However, this is a great opportunity for members of both organizations to bond stronger ties. I miss meeting with the HSSFC very much, in particular the camaraderie that ensues with academics in other disciplines, and the wonderful eclectic displays of books in the exhibition hall. I look forward to meeting again with the HSSFC at the Université de Laval in 2001.

In another WMY 2000-related activity, we have received an invitation from the American Mathematical Society (AMS), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) to send a representative to the opening banquet of the Joint Mathematics Meeting held January 18, 2000 in Washington, D.C. Our Society has also been invited to prepare a resolution celebrating WMY 2000 for inclusion in the banquet program.

Since my last message, I have been fortunate to have spoken on various topics in history of mathematics to the New England Advanced Placement Mathematics Teachers at Milton Academy in Massachusetts, at the fall meeting of the MAA Southern California Section at Pepperdine University in Malibu, at the MAA Eastern Pennsylvania and Delaware Section at Lehigh University in Bethlehem, Pennsylvania, and at the MAA Northern California Section Meeting at Ohlone College in Fremont, California. I never cease to be impressed at the number of attendees genuinely interested in learning more about the history of the subject. I especially enjoy listening to others lecture and gleaning bits of mathematics and history from their talks.

Last but far from least, I look forward to seeing many of you at our annual meeting in Toronto this summer, which will be part of the second CSPM-BSHPM Joint Meeting held at the University of Toronto from July 15 to 17.

Exercises (Cont. from p. 6)

Notes:

(1) Angelo, Thomas A. and K. Patricia Cross. (1993). *Classroom Assessment Techniques. A Handbook for College Teachers*. 2nd Ed. San Francisco: Jossey-Bass Publishers, pp. 188-92.
(2) Lakatos, Imre. (1976). *Proofs and Refutations. The Logic of Mathematical Discovery*. J. Worrall and E. Zahar, eds. Cambridge: Cambridge University Press.

Madeline Muntersbjorn <mmunter@uoft02. utoled.edu>, Department of Philosophy, University of Toledo, Toledo, OH 43606 studies the role representational innovations play in the growth of mathematical knowledge. Her recent work explores the development of analytic geometry in the first half of the 17th century, with an emphasis on the "pre-calculus" papers of Fermat.

How I learned to love the history of mathematics ...

One night last May, during the Society's Annual Meeting in Ottawa, the seven members whose names appear below stood on a street corner and fell to swapping personal histories of their involvement with mathematics and/or its history and philosophy — how they learned to love the subject, how it came to be part of their professional lives, and so on. Eventually someone suggested that it would be fun to share these accounts with other members, through the newsletter; hence the snippets of autobiography that follow. There must be many other such tales out there; feel free to submit yours, in roughly the spirit and at roughly the length, of these first examples. If we collect another set in this way, we'll run them in a later issue.

Fran Abeles

Game. Dorothy Geddes taught mathematics at Hunter College High School, and I had the good fortune to be one of her students. Miss Geddes, as she was called then, loved mathematics as a discipline, and with no vocational motives attached, she silently communicated that love. Oh yes, she also taught so well that, astonishingly, I scored 100% on both the geometry and solid geometry/trigonometry (including spherical trigonometry) New York State Regents examinations. I was hooked.

Set. At Barnard College, Columbia University, I found that mathematics course requirements, producing solutions to problems and proofs of theorems, neatly balanced the heavy reading and term-paper loads of other courses. When I decided to double-major, I chose mathematics as one of them.

Matchpoint. A graduate course at Columbia, in historiography, awakened a dormant interest in history. (Years earlier, I had scored the maximum 800 points in the SAT history subject test of the College Entrance Examination Board.) The University didn't offer the option of studying the history of mathematics, so I just read historical material in connection with the courses I was taking. However, I was able to elect an historical topic for the dissertation.

Match. A master's degree in computer science from Stevens Institute of Technology allows me to teach computer science courses at Kean University. Except for mathematical logic and an occasional graduate mathematics course, computer science is all I do teach now. But I hope to teach a graduate course in the history of 19th- and 20th-century mathematics in the fall 1999 term — if there are enough students to provide sufficient enrolment.

Pat Allaire

I view myself as an accidental historian, but then I'm an accidental mathematician as well.

Though elementary school and junior high I did arithmetic and algebra competently but passionlessly. Then came Euclidean geometry. Suddenly I was in love. That statement A implies statement B via 10 other statements, a notion that never struck me in algebra, was a magnificent one. Now equipped with the ever-cool A - - >B, I charged through several more levels of algebra as well as trig and solid geometry. However, I had a second love - foreign languages, and I took every high school course available in that field as well.

I enrolled in college unsure of which area would be my major. But on the day of freshman registration, an advisor bellowed to the crowd: "Be sure you pick up the correct schedule: there's one for math majors, another for everyone else." I had to make an instant decision. I reached for the math-major schedule and never looked back.

Lots of years later, having completed the required courses for a doctorate, I was studying for the oral exams and pondering the choice of an area in which to write my dissertation. I'd had a few history-of-math courses at various levels, and found the area intriguing. I noticed an announcement of the NSF/MAA Institute on the History of Mathematics, but put it aside because the exams were imminent. However, a colleague who was aware of my interest in history, saw the ad and nagged me into applying. The three weeks of historical immersion did it — I was hooked. And when Helena Pycior presented the first of her five lectures at the Institute, I knew I was home: algebra in Britain in the first half of the nineteenth century was to be the focus of my research.

Hardy Grant

One of Rodney Dangerfield's best jokes — so, okay, call me a lowbrow — is about the guy who's *so* old that in his school days they couldn't teach history: nothing had happened yet. *I'm* so old that at university I couldn't study history of mathematics — the subject's professional and curricular status was then still marginal at best. Not that I ever *wanted* to — the idea never entered my head; which is perhaps the more surprising as I was already established in the lifelong joy and dilemma of squarely straddling Lord Snow's notorious cultural divide. I always loved history; two of my four "desert island" authors (Will Durant and Joseph Needham) have that distinction just because they dared to write history on the Grand Scale. But I always loved mathematics too, and since one has to specialize in *something*, I made that my major, and I never regretted the choice.

I was duly ensconced in a university department (York, in Toronto) before I was obliged to concede once and for all that I have no talent whatever for original research in mathematics. But meanwhile I found that by pure luck I had stumbled onto a scene graced by several people who were (and are) very interested — and in some cases very active — in the subject's history: Israel Kleiner, Trueman MacHenry, Martin Muldoon, Pinayur Rajagopal, Abe Shenitzer. Abe and Israel in particular prodded and supported me in ways not easy to acknowledge, let alone to repay. After awhile our little cabal contrived official approval for an undergraduate course (eventually third-year) in York's Humanities Division on the history and cultural influence of mathematics, and when the dust settled I wound up as principal instructor. I scrambled to organize and supplement a lot of desultory reading, realized how much I love the stuff, taught it for 17 years, joined the CSHPM, and now propose to live happily ever after. Moral? None at all that I can see, unless the banal observation that if you're going to insist on sitting astride disciplinary boundaries it is very agreeable to find a seat that fits your butt.

Jacques Lefebvre

When a young man, I saw myself as a future philosopher. But I thought it was better to get a training and a knowledge of something else before, and I went into a mathematics and physics undergraduate program (was I unknowingly obeying the Platonic dictum?).

I never got to formal philosophical studies and started teaching mathematics after my Honours B.Sc. I did my graduate studies in mathematics after the foundation of Université du Québec à Montréal (UQAM), in 1969, had closed several schools, including the "collège classique" where I taught.

It was only in the mid-70s that history of mathematics became present in my intellectual and professional life, first through introductory seminars held among colleagues. I was then assigned to set up and deliver courses in the history of mathematics, at the undergraduate level and afterwards at the graduate one. How does a not-yet-too-well-learned-in-the-field teacher manage to do that without distress or even total failure? By choosing a solid textbook, at that time the one by Boyer.

A period of about eight years of full-time administrative duties intervened. In 1987 I went back to normal professorial life and to ... the history of mathematics (and of science, more generally and loosely), which is now my main professional field of interest and activity. Ironically I have lately been investigating the role given to mathematics by some 17th-century philosophers — so philosophy is still present.

On the whole, I have been involved with the history of mathematics a bit late and intermittently. My relation to it has been partly accidental but certainly corresponds to a deeply rooted personal interest in the nature and development of knowledge.

Gregory H. Moore

My interest in history of mathematics (hereafter HOM) was sparked during the summer of 1967, when I took a course in HOM given at the University of California at Berkeley by Ken May, already eminent in the field. I did my major paper on Galois. At the end of the course he told me that if I ever wished to do graduate work in HOM, I should come to the University of Toronto and study with him. After a year teaching high school and then two years teaching at an experimental elementary school in Berkeley, I arrived in Toronto in August 1970. During the next few years, May had the principal group in Canada of graduate students in HOM. It included, besides myself: Philip Enros, Charles Jones, Steve Regosci, Hank Tropp, and, later, Craig Fraser. May encouraged me to work on Zermelo, and reinforced my interest in the Axiom of Choice (the subject of my dissertation and of my first book).

It was a striking period. In 1972 May completed his bibliography of HOM, and was starting *Historia Mathematica*, whose first issue appeared in 1974. That same year he had a heart attack, and asked me to take over teaching his HOM course. In December 1977 he had a second heart attack, and died. Again I took over his HOM course. Several of his doctoral students (Enros, Fraser, Jones, and myself) were forced to find another supervisor for our doctoral dissertations; in my case, Joseph Dauben was brought in for this purpose. Nevertheless, Ken May had molded my way of doing HOM with his emphasis on evidence (i.e. labeling conjecture as such) and on seeing the historical material afresh. I am eternally indebted to him.

Johnathan Seldin

I was first attracted to mathematics by an excellent high school teacher that I had: James D. Bristol. I was in his advanced placement class at Shaker Heights High School (in Ohio, a suburb of Cleveland) for three years, 1957-1960. In those three years, I went from algebra through more than the first year of calculus. Before I was in his class, I knew I was interested in science; after being in that class, I knew that I wanted to specialize in mathematics.

I have always had an interest in history, and I have always enjoyed reading history for relaxation. My attention was directed to the history of mathematics in the early 1980s when, for four years, I taught the upper-level undergraduate course in geometry at Concordia University. I had been concerned about the fact that many of my students were having trouble understanding what mathematical proofs are all about. While teaching this geometry course, I realized that the history of the mathematical notion of proof goes back to the ancient Greeks, and I started looking into that history. I soon became convinced that looking at this history could help clarify this notion for modern students. One of the results of this research was the first two papers I presented to this Society for History and Philosophy of Mathematics: "Reasoning in elementary mathematics" in 1989 and "From exhaustion to modern limit theory" in 1990.

Jim Tattersall

I was a recipient of good although latent mathematical genes courtesy of my uncle Eugene who had a good knowledge of the subject. My only memories of him are that at one time he possessed the only good set of plans for the underground gas lines in Fall River, Massachusetts, a city immortalized by Lizzie Borden, and that he was a notoriously bad driver. I attended elementary schools in New York and Virginia but missed the greater part of the third and seventh grades.

Up to and during secondary school in Fairfax County, Virginia I was involved with sports both as a player and one fascinated with how descriptive statistics were used to compare players and teams. I enjoyed algebra but did not distinguish myself in my first two encounters with the subject until I encountered Mrs. Stine, an outstanding no-nonsense teacher who demanded rigor and absolute knowledge of the basics.

One evening my father took me to the apartment of a mathematician who worked for the government. The minute I saw the den with a desk surrounded by shelves of mathematics texts I knew I had found my profession. In secondary school, over a guidance counselor's objections, I took courses in plane geometry, solid geometry, probability, trigonometry, and calculus with Mrs. Stine, who worked on a Master's degree at American University evenings and shared her graduate school experiences with us on numerous occasions. Her students received scholarships to college and my graduation she awarded me the mathematics medal albeit mainly for persistence and inquisitiveness since my grades in no way indicated a career in the subject. My interest in history goes back to secondary school when I found myself the only person in the class willing to defend the North's position in the U.S. Civil War.

I attended the University of Virginia where although topology ruled undergraduates were given a steady diet of real and complex analysis. I worked as a grader for two years and gave exams when professors were away at conferences. When I attended graduate school, as luck would have it, one of the graduate assistants failed to show up for work and I was assigned their classes thereby beginning my career as a teacher. Most of my master's level courses were algebraic in nature although my doctoral thesis was in combinatorial topology.

I took a position at Providence College in Rhode Island and spent many an afternoon in the Brown University libraries reading history and mathematics. My initial historical quest concerned the mathematics Thomas Jefferson, the founder of the University of Virginia, understood and how he made use of it. I spent the greater part of a sabbatical leave in 1985 reviving my mathematical skills attending Bela Bollobas' combinatorial seminar at Cambridge University. I presented my contribution to the seminar one frosty February afternoon and must have overly stressed the historical aspects of my work for the consensus was that I contact D.T. Whiteside and Piers Bursil-Hall and spend my remaining time at Cambridge studying the history of mathematics. I took the sage advice, never regretted it, and the rest is history.

Winter Joint Meetings of the AMS/MAA

Members of our society figured prominently in the History of Mathematics sessions at the joint mathematics meetings of the AMS and MAA held in San Antonio this past January. Most notably John Fauvel presented an MAA invited address, "The History of Mathematics and its Future." Karen Parshall and Victor Katzorganized an AMS/MAA Special Session on the History of Mathematics and Victor was a presenter at an MAA special presentation on the use of history in the teaching of mathematics. Other members spoke at the various sessions.

In addition to talks and courses, history appeared in other forms. Several times a day the film "Challenge in the Classroom" focussing on R. L. Moore and the Moore Method could be seen in the days-long MAA Special Presentation dealing with the legacy of Moore. Evening entertainment, too, was historically based. A dramatic presentation on the mathematics of Lewis Carroll played to a full house on Wednesday evening, while Thursday evening's musical presentation, "Eine Kleine (Mathematische) Nachtmusic", considered the mathematical basis of the various tunings that have been used over the centuries.

Following is a listing of members of the society who spoke at the meetings and the titles of their talks. If we have inadvertently missed anyone, please contact the editor and you will be included in the next Bulletin. Also if any of you feel that the material presented would be appropriate for this Bulletin, please submit same to the editors and we will consider it for publication in a future issue.

Patricia R. Allaire, Foundation issues in the 19th century British school; negative and complex numbers.
Thomas Archibald, Charles Hermite and Franco-German relations in mathematics, 1870 - 1885.
Eisso J. Atzema, Some aspects of visualization in 19th-century mathematics and its teaching.
E. Kreyszig, On the development of certain numerical methods.
Albert C. Lewis, R.L. Moore and Inovations in Mathematics Education.
Glen R. Van Brummelen, The Role and Development of Analysis and Synthesis in Ancient Greece and Medieval Islam.

Book Review

The Calendar, by D.E. Duncan, published (1998) by Fourth Estate (London)

Reviewed by **Peter L. Griffiths**

The Fourth Estate continues its theme of popularising the sciences with *The Calendar* by the American author D.E. Duncan. The author starts by describing the problems confronting Roger Bacon (c. 1267) in trying to explain the need for calendar reform to Pope Clement IV. He then shows (pp. 17-19) that the lack of synchronisation between the movements of the sun and moon was something of concern to the earliest civilisations. However it should be pointed out that the early Babylonians were interested in the signs of the Zodiac not primarily for astrological reasons but because of the regularity of the movement of these recognisable constellations during the night and indeed throughout the year. One might offer an alternative to Duncan's ideas (pp. 19-20) about the significance of the hour by pointing out that at the start of the night there would be six signs of the Zodiac displayed across the sky, and at the end of the night the other six would be visible. Divide the time of the night by this figure of twelve, and the result is one hour. The mathematical logic is not impeccable, but it is credible. The division of the hour into 60 minutes, and the minute into 60 seconds, would have been inspired by the old Babylonian sexagesimal system.

The New Testament. On pages 67-68, the author gives various fanciful reasons for the virtual omission of satisfactory dates from the New Testament. It should however be pointed out that unlike the works of Josephus (A.D. 37-c. 100), which contain numerous dates, the New Testament contains only one year date (in Luke 3:1); this reflects the different motives of the writers. Josephus's purpose was to record the facts of history as best he could, whereas the motive of the writers of the New Testament was to create a new religion, particularly for the Jews, involving a replacement of Passover by a new festival commemorating the crucifixion and resurrection of Jesus Christ. The result is that in the New Testament there are many references to the time of the Passover but none to its actual year.

Stating the Year. The main system for stating the year was to specify the year of the reign of the country's ruler. One variation on this was the Roman *ab urbe condita*, meaning "from the foundation of the city". The other variation was the Christian system, invented by Diogenes Exiguus (c. 531 A.D.). Duncan indicates (p. 100) that adopting the date of Jesus's incarnation was the main theological motive for the change to Diogenes's system from the existing one based on the accession of Diocletian in 284. Dionysius Exiguus recognised 25 December as the birthday of Jesus Christ, but theologically he was more interested in the date of the incarnation, which could have occurred at any time within the previous nine months. To be on the safe side he decided that 25 March should be recognised as the date of the incarnation, and hence 25 March of 1 A.D. would become the first day of the Christian era, and 25 March would be the first day of the ecclesiastical year. This would be in contrast to the Jewish calendar, whereby the New Year was always 163 days after Passover.

Duncan seems to contradict himself regarding the start of the Julian calendar. On page 46 he mentions 1 January, but on page 111 it becomes 25 December. The significance of 25 December was not recognised until much later, about 336 A.D., when it was included as the feast of the birth of Jesus Christ in an almanac of the church's holy days. (See the entry "Christmas" in *The Oxford Dictionary of the Christian Church*, edited by Cross and Livingstone.) An important historical fact completely omitted by the author is that in most Protestant countries 25 March was the first day of the year until 1752, when, along with the loss of the eleven days, it was replaced by 1 January. As the author rightly says on the back cover, the loss of the eleven days gave rise to 5 April as being the end of the British tax year, but he omits to mention that the actual difference between 25 March and April 6 (the first day of the new tax year) is 12 days not 11.

On pages 101-2 the allegation is made that the first mention of "Before Christ" occurs in 1627. In fact, the original Latin version of Bede's *Ecclesiastical History of the English People* expresses dates before 1 A.D. as "ante incarnationem Domini Nostri", so that the concept if not the actual wording should be attributed to Bede. On page 274 the diagram of the Earth's tilt in its elliptical orbit round the Sun is unconventional but not incorrect. The direction of the tilt really depends on whether the Sun's view of the Earth in spring or in autumn is preferred. The author seems to prefer the autumn view; readers should have been told this. On page 143 Duncan professes to know nothing about the obscure saints Linus and Cletus, but as any Catholic reference book would have told him, these are the names of the second and third Popes. Linus is mentioned in Timothy 4:21.

On pages v-vii, under the heading "Calendar Index", Duncan gives a useful outline of the history, showing that the ancient Egyptians succeeded in measuring the solar year as 365 1/4 days. One of the major problems since then has been the mathematical reconciliation of the solar and lunar calendars. Diogenes Exiguus's Christian system of stating the year has been generally accepted, even by most non-Christians. This however has not prevented disputes within Christendom over the dating of Easter, the start of the New Year, and the introduction of the Gregorian calendar — initially by the Roman Catholic states in 1582 and later by most of the rest of the world, as vividly described by the author in Chapter 14.

In spite of its shortcomings, this book is well structured, and after a thorough revision with the assistance of a knowledgeable referee could become a useful and entertaining work of reference.

References:
F.L. Cross and E.A. Livingstone, eds., *Oxford Dictionary of the Christian Church*, Oxford University Press, 1974.
Peter L. Griffiths, *Who Wrote the New Testament and Why?* Minerva Press, 1994.

"Ever since the mathematicians began to penetrate into the innermost of feminine sanctuaries and ... to bring with them the terminology of a science as solid and serious as mathematics, we hear that Cupid's empire is rapidly crumbling, and that no one talks now of anything but problems, corollaries, theorems, right-angles, obtuse angles, rhomboids, and so on. ... [One] young lady positively refused a perfectly eligible suitor simply because he had been unable, within a given time, to produce any new idea about 'squaring the circle'." — *Journal des savants*, 4 mars 1686

PERSONALS

Pat Allaire received a PSC-CUNY 1999-2000 research award. The project for which the award was granted is "A Search for the Foundations of Calculus through British Symbolical Algebra Emphasizing the Contributions of Duncan F. Gregory." In this project, she and Rob Bradley will prepare an updated, annotated edition of portions of an 1841 Gregory text.

Marcia Ascher's book *Ethnomathematics: A Multicultural View of Mathematical Ideas* (paper ed. Chapman & Hall / CRC, 1994) is now available also in French, as *Mathématiques d'ailleurs* (Editions du Seuil, Paris; paper). The translation and 20-page afterword are by Karine Chemla and Serge Pahaut.

Abe Shenitzer has completed translations of (i) Detlef Laugwitz's intellectual biography of Riemann, (ii) Bashmakova and Smirnova, *The Beginning and Evolution of Algebra*, and (iii) Hadamard's essay "Non-Euclidean Geometry in the Theory of Automorphic Functions". The Laugwitz book is about to appear; the others have been sent to the MAA and AMS respectively.

Sylvia Svitak is pleased to announce (i) the fall 1998 publication by Addison Wesley Longman of the preliminary editions of textbooks she co-authored with fellow members of the Consortium for Foundation Mathematics and (ii) that she is a recipient of a 1999-2000 PSC-CUNY research award for her project, "Functional Models for Understanding Factor Analysis." The texts, *Mathematics in Action: An Introduction to Algebraic, Graphical, and Numerical Problem Solving*, and *Mathematics in Action: Algebraic, Graphical, and Trigonometric Problem Solving,* take the approach that students best learn mathematics in meaningful and realistic problem-solving contexts. The Consortium is currently preparing the first editions, which will appear in August 2000. Sylvia and her co-authors hope that their texts will "make history" and change the way mathematics is taught to college students in need of basic mathematical concepts and skills. Sylvia expects to use her award towards her goal to publish a resource manual for researchers, teachers, and students that addresses the issues concerning the proper use of factor analysis in scientific research.

David E. Zitarelli has published *A Collaborative Approach to Mathematics* (Raymond-Reese Book Co., Wyncote, PA, 1999). This is a textbook for future K-8 teachers that contains 31 modules on number systems, real numbers, set theory. Each module is placed in its historical or multicultural setting.

Brazilian Society for History of Mathematics founded

The Brazilian Society for History of Mathematics (Sociedade Brasileira de Historia da Matematica / SBHM) was founded in March of this year in Vitoria, Espirito Santo, during the III National Seminar in History of Mathematics. Long-time CSHPM member Ubiratan ("Ubi") D'Ambrosio was honoured by election as President; Circe Dynikov is Vice-President, Sergio Nobre is Secretary-General, Marcos Vieira Teixeira is Treasurer, John Fossa is Secretary. "We all hope", says Ubi, "the SBHM will stimulate the area in Brazil and will have an international presence, in close cooperation with other societies and historians of mathematics around the world".

Secretary-Treasurer's Report, 1998

1998 was a stable year for the Society. Our cross-membership arrangements with the CSHPS and BSHM have been functioning for one year, and membership figures remain about constant after five years of rapid growth (see chart below). The administrative portion of the SSHRC grant was completely phased out in 1998; nevertheless the Society managed to operate on roughly an even keel. Our year-end balance has benefited from several years of substantial surpluses, giving us a satisfying "cushion".

Financial Statement (must be approved at the Annual General Meeting)

Credit		*Debit*	
SSHRC grant	$0.00	Publications (*Bulletin*, *Proceedings*)	$1111.58
Membership dues	$11083.45	HSSFC dues	$922.00
Other income (currency exchange, sale of *Proceedings*, etc.)	$293.08	*Historia Mathematica*	$3930.83
		Philosophia Mathematica	$1897.15
		Conference expenses (keynote)	$310.00
		Travel	$133.42
		Sponsorship, CUMC conference	$100.00
		Misc. (postage/copying, bank charges, etc.)	$139.99
TOTAL:	$11376.53	TOTAL:	$8554.97

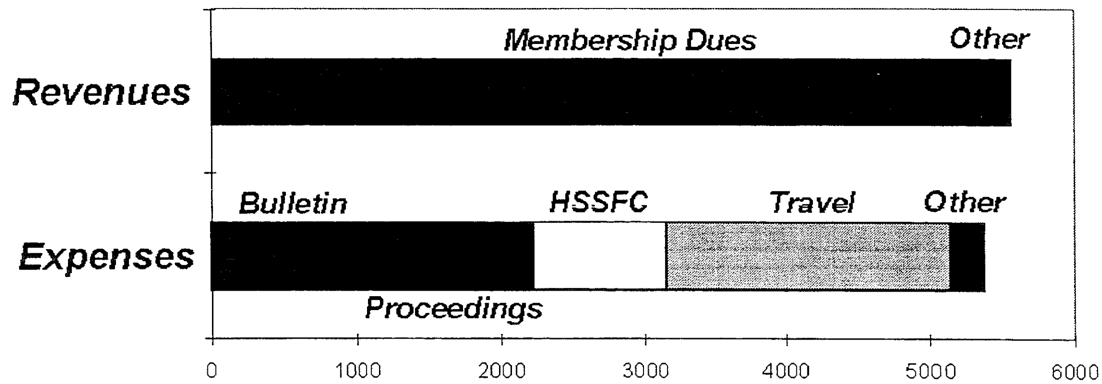
Surplus:	**$2831.56**
Amount carried forward (1997):	$6753.35
YEAR-END BALANCE:	**$9584.91**

Revenues and Expenses:

While this year's statement shows a healthy surplus, it is misleading: two major expenses for 1998, totaling about $2650, were not cleared until early 1999. It is nevertheless reassuring that we have been able to emerge from the loss of SSHRC administrative funding with a healthy bottom line.

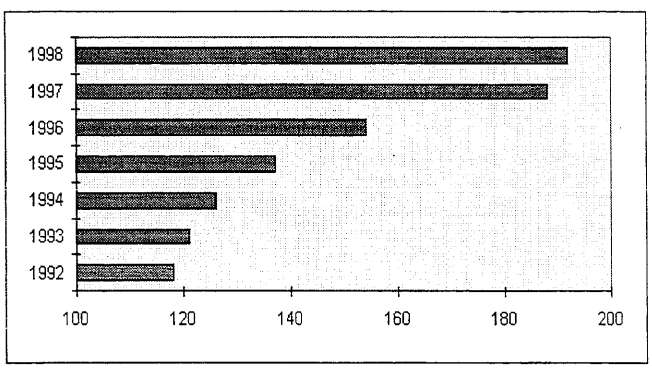
Other than the inclusion of the above two expenses in the 1999 statement, I do not expect major changes to occur in 1999. Hence, I project a deficit roughly equal to $2650.

Two major items in the above statement (*Historia Mathematica* and *Philosophia Mathematica* subscriptions) are "in-out" budget items, and may obscure what our dues money is actually spent on. The graph below represents revenues and expenses for 1998, removing these two items and adding the two major expenses noted above. It should give you a better idea of the allocation of Society funds.



Membership totals:

After several years of substantial growth, the Society's membership grew marginally in 1998, from 188 to 192. Early figures for 1999 suggest that we should exceed 200 members for the first time. The graph shows our recent membership history. (1997 was the first year of our cross-membership arrangements with the BSHM and CSHPS.)



SSHRC Travel Subsidies

SSHRC continues to provide funds for members' travel to our annual meeting. The amounts were reviewed this year. Due to our rapid rise in membership and increased funding for the program, our funding for 1999 has increased from $1514 to $3480! Although travel subsidies depend on the number of applications received, the larger "pot" will hopefully defray our travel expenses more than in past years.

Glen Van Brummelen, Secretary-Treasurer

Web Review: Earliest Known Uses of Mathematical Words and Symbols

(Jeff Miller)

http://members.aol.com/jeff570/mathword.html
http://members.aol.com/jeff570/symbol.html

Curiosity about the language of mathematics can be found even among our least-interested students. Many of us have been asked about the origins of symbols such as the sign for equality, or the word *integral*. I expect almost all of us have been asked why *m* is used as the symbol for the slope of a line![1] Answers to these questions can help to demystify the nature of mathematical communication. Robert Recorde's explanation of his use of two parallel lines for equality (*because no two things can be more equal*) usually gets a glimmer of life even from the students who hide at the back of the classroom. However, sources of this sort of information are scattered, incomplete, and difficult to find.

Jeff Miller's pair of sites on the first appearances of mathematical words and symbols is a fairly comprehensive attempt to gather information on these topics. Beginning with the standard print sources *A History of Mathematical Notations* by Florian Cajori and the *Oxford English Dictionary*, Miller is building his collection using the resources of historians of mathematics worldwide. He has been able to update and correct a number of entries, and add others, partly through the assistance of members of the two e-mail lists devoted to the history of mathematics. The result is a thorough, detailed reference that is not only a nice place to go to answer students' questions, but also an entertaining site to browse.

These sites exemplify what is best about the Internet. It's hard to imagine that a high school teacher, even one with Miller's obvious dedication and enthusiasm, would have been able to convince a book publisher of the worthiness of his endeavour. Nor could he have achieved such results without the fast and diverse collaboration of over 30 scholars (so far). Finally, the site is constantly evolving as new information is brought to Miller's attention; many of the pages I read had been updated only days previously. Try that with a book!

Don't wait for a student's question to visit these sites. You will find many tidbits to enliven your lectures by introducing a bit of history. If you are aware of information that would correct or improve an entry, I'm sure that Jeff would be delighted to hear from you. This increasingly communal project will be a worthy contribution to future research, and a notable legacy from the first generation of the Internet.

"The most decisive change in the tenor of Western intellectual life since the seventeenth century is the submission of successively larger areas of knowledge to the modes and proceedings of mathematics." — George Steiner, *Language & Silence*

[1] For information on this mystery, visit http://forum.swarthmore.edu/dr.math/faq/faq.mforslope.html.

The following have recently joined the Society. A warm welcome to all!

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